Microwave Devices and Circuits

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Chapter 10

Microwave Crossed-Field Tubes (*M* Type)

10-0 INTRODUCTION

In the previous chapter, several commonly used linear-beam tubes were described in detail. In these tubes, the dc magnetic field that is in parallel with the dc electric field is used merely to focus the electron beam. In crossed-field devices, however, the dc magnetic field and the dc electric field are perpendicular to each other. In all crossed-field tubes, the dc magnetic field plays a direct role in the RF interaction process.

Crossed-field tubes derive their name from the fact that the dc electric field and the dc magnetic field are perpendicular to each other. They are also called M-type tubes after the French TPOM (*tubes à propagation des ondes à champs magnétique:* tubes for propagation of waves in a magnetic field). In a crossed-field tube, the electrons emitted by the cathode are accelerated by the electric field and gain velocity, but the greater their velocity, the more their path is bent by the magnetic field. If an RF field is applied to the anode circuit, those electrons entering the circuit during the retarding field are decelerated and give up some of their energy to the RF field. Consequently, their velocity is decreased, and these slower electrons will then travel the dc electric field far enough to regain essentially the same velocity as before. Because of the crossed-field interactions, only those electrons that have given up sufficient energy to the RF field can travel all the way to the anode. This phenomenon would make the M-type devices relatively efficient. Those electrons entering the circuit during the accelerating field are accelerated by means of receiving enough energy from the RF field and are returned back toward the cathode. This back-bombardment of the cathode produces heat in the cathode and decreases the operational efficiency.

In this chapter, several commonly used crossed-field tubes such as magnetrons, forward-wave crossed-field amplifiers (FWCFAs), backward-wave crossed-field amplifiers (BWCFAs or amplitrons), and backward-wave crossed-field oscillators (BWCFOs or carcinotrons) are studied.

Cylindrical magnetron: The cylindrical magnetron was developed by Boot and Randall in early 1940.

Coaxial magnetron: The coaxial magnetron introduced the principle of integrating a stabilizing cavity into the magnetron geometry.

Voltage-tunable magnetron: The voltage-tunable magnetron has the cathode-anode geometry of the conventional magnetron, but its anode can be tuned easily.

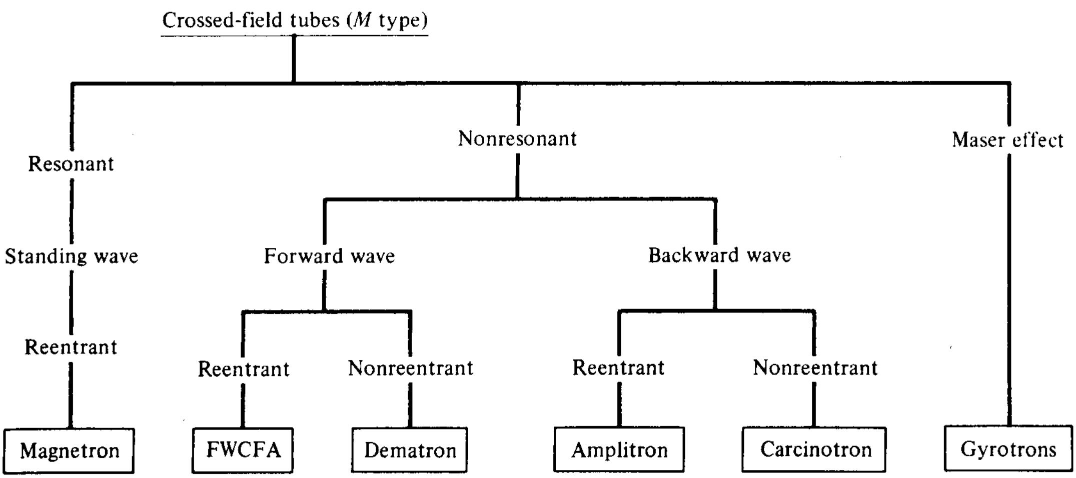
Inverted magnetron: The inverted magnetron has the inverted geometry of the conventional magnetron with the cathode placed on the outside surrounding the anode and microwave circuit.

Forward-wave crossed-field amplifier (FWCFA): The forward-wave crossed-field amplifier is also called the M-type forward-wave amplifier.

Backward-wave crossed-field amplifier (BWCFA): The backward-wave crossed-field amplifier was developed by Raytheon Company in 1960 and its trade name is Amplitron. It is a broadband, high-power, high-gain, and high-efficiency microwave tube, and it has many applications such as in airborne radar systems and spaceborne communications systems.

Backward-wave crossed-field oscillator (BWCFO): In 1960, the French Company developed a new type of crossed-field device in which an injection gun replaced the conventional cylindrical cathode of the magnetron. Its trade name is Carcinotron, and it is also called the *M-type backward-wave oscillator*.

All of these crossed-field electron tubes are tabulated in Table 10-0-1.



10-1 MAGNETRON OSCILLATORS

Hull invented the magnetron in 1921 [1], but it was only an interesting laboratory device until about 1940. During World War II, an urgent need for high-power microwave generators for radar transmitters led to the rapid development of the magnetron to its present state.

All magnetrons consist of some form of anode and cathode operated in a dc magnetic field normal to a dc electric field between the cathode and anode. Because of the crossed field between the cathode and anode, the electrons emitted from the cathode are influenced by the crossed field to move in curved paths. If the dc magnetic field is strong enough, the electrons will not arrive in the anode but return instead to the cathode. Consequently, the anode current is cut off. Magnetrons can be classified into three types:

1. *Split-anode magnetron:* This type of magnetron uses a static negative resistance between two anode segments.
2. *Cyclotron-frequency magnetrons:* This type operates under the influence of synchronism between an alternating component of electric field and a periodic oscillation of electrons in a direction parallel to the field.
3. *Traveling-wave magnetrons:* This type depends on the interaction of electrons with a traveling electromagnetic field of linear velocity. They are customarily referred to simply as *magnetrons*.

Negative-resistance magnetrons ordinarily operate at frequencies below the microwave region. Although cyclotron-frequency magnetrons operate at frequencies in microwave range, their power output is very small (about 1 W at 3 GHz), and their efficiency is very low (about 10% in the split-anode type and 1% in the single-anode type). Thus, the first two types of magnetrons are not considered in this text. In this section, only the traveling-wave magnetrons such as the cylindrical magnetron, linear (or planar) magnetron, coaxial magnetron, voltage-tunable magnetron, inverted coaxial magnetron, and the frequency-agile magnetron will be discussed.

10-1-1 Cylindrical Magnetron

A schematic diagram of a cylindrical magnetron oscillator is shown in Fig. 10-1-1. This type of magnetron is also called a *conventional magnetron*.

In a cylindrical magnetron, several reentrant cavities are connected to the gaps. The dc voltage V_0 is applied between the cathode and the anode. The magnetic flux density B_0 is in the positive z direction. When the dc voltage and the magnetic flux are adjusted properly, the electrons will follow cycloidal paths in the cathode-anode space under the combined force of both electric and magnetic fields as shown in Fig. 10-1-2.

Equations of electron motion. The equations of motion for electrons in a cylindrical magnetron can be written with the aid of Eqs.(1-2-5a) and (1-2-5b) as

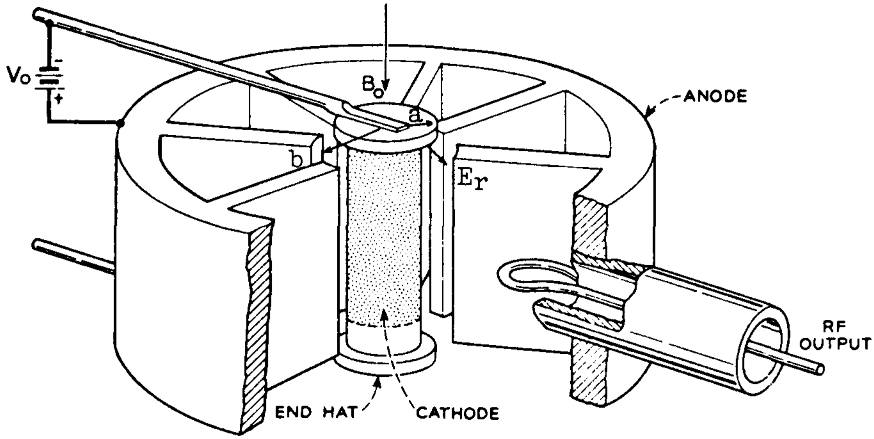


Figure 10-1-1 Schematic diagram of a cylindrical magnetron.

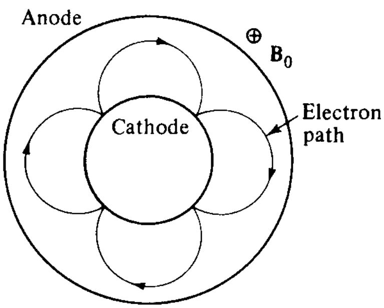


Figure 10-1-2 Electron path in a cylindrical magnetron.

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_z\frac{d\phi}{dt} \tag{10-1-1}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt} \tag{10-1-2}$$

where $\frac{e}{m} = 1.759 \times 10^{11}$ C/kg is the charge-to-mass ratio of the electron and $B_0 = B_z$ is assumed in the positive z direction.

Rearrangement of Eq. (10-1-2) results in the following form

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z r\frac{dr}{dt} = \frac{1}{2}\omega_c\frac{d}{dt}(r^2) \tag{10-1-3}$$

where $\omega_c = \frac{e}{m}B_z$ is the cyclotron angular frequency. Integration of Eq. (10-1-3) yields

$$r^2\frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 + \text{constant} \tag{10-1-4}$$

at $r = a$, where a is the radius of the cathode cylinder, and $\frac{d\phi}{dt} = 0$, constant $= -\frac{1}{2}\omega_c a^2$. The angular velocity is expressed by

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{r^2}\right) \tag{10-1-5}$$

Since the magnetic field does no work on the electrons, the kinetic energy of the electron is given by

$$\frac{1}{2}m\mathcal{V}^2 = eV \tag{10-1-6}$$

However, the electron velocity has r and ϕ components such as

$$\mathcal{V}^2 = \frac{2e}{m}V = \mathcal{V}_r^2 + \mathcal{V}_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 \tag{10-1-7}$$

at $r = b$, where b is the radius from the center of the cathode to the edge of the anode, $V = V_0$, and $dr/dt = 0$, when the electrons just graze the anode, Eqs. (10-1-5) and (10-1-7) become

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right) \tag{10-1-8}$$

$$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m}V_0 \tag{10-1-9}$$

Substitution of Eq. (10-1-8) into Eq. (10-1-9) results in

$$b^2\left[\frac{1}{2}\omega_c\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m}V_0 \tag{10-1-10}$$

The electron will acquire a tangential as well as a radial velocity. Whether the electron will just graze the anode and return toward the cathode depends on the relative magnitudes of V_0 and B_0. The *Hull cutoff magnetic equation* is obtained from Eq. (10-1-10) as

$$B_{0c} = \frac{\left(8V_0\frac{m}{e}\right)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)} \tag{10-1-11}$$

This means that if $B_0 > B_{0c}$ for a given V_0, the electrons will not reach the anode. Conversely, the cutoff voltage is given by

$$V_{0c} = \frac{e}{8m}B_0^2 b^2\left(1 - \frac{a^2}{b^2}\right)^2 \tag{10-1-12}$$

This means that if $V_0 < V_{0c}$ for a given B_0, the electrons will not reach the anode. Equation (10-1-12) is often called the *Hull cutoff voltage equation*.

Example 10-1-1: Conventional Magnetron

An X-band pulsed cylindrical magnetron has the following operating parameters:

Anode voltage:	$V_0 = 26$ kV
Beam current:	$I_0 = 27$ A
Magnetic flux density:	$B_0 = 0.336$ Wb/m^2
Radius of cathode cylinder:	$a = 5$ cm
Radius of vane edge to center:	$b = 10$ cm

Compute:

a. The cyclotron angular frequency
b. The cutoff voltage for a fixed B_0
c. The cutoff magnetic flux density for a fixed V_0

Solution

a. The cyclotron angular frequency is

$$\omega_c = \frac{e}{m}B_0 = 1.759 \times 10^{11} \times 0.336 = 5.91 \times 10^{10} \text{ rad}$$

b. The cutoff voltage for a fixed B_0 is

$$V_{0c} = \frac{1}{8} \times 1.759 \times 10^{11} (0.336)^2 (10 \times 10^{-2})^2 \left(1 - \frac{5^2}{10^2}\right)^2$$
$$= 139.50 \text{ kV}$$

c. The cutoff magnetic flux density for a fixed V_0 is

$$B_{0c} = \left(8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}}\right)^{1/2} \left[10 \times 10^{-2}\left(1 - \frac{5^2}{10^2}\right)\right]^{-1}$$
$$= 14.495 \text{ mWb/m}^2$$

Cyclotron angular frequency. Since the magnetic field is normal to the motion of electrons that travel in a cycloidal path, the outward centrifugal force is equal to the pulling force. Hence

$$\frac{m\mathcal{V}^2}{R} = e\mathcal{V}B \tag{10-1-13}$$

where R = radius of the cycloidal path
$\mathcal{V}$ = tangential velocity of the electron

The cyclotron angular frequency of the circular motion of the electron is then given by

$$\omega_c = \frac{\mathcal{V}}{R} = \frac{eB}{m} \tag{10-1-14}$$

The period of one complete revolution can be expressed as

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{eB} \tag{10-1-15}$$

Since the slow-wave structure is closed on itself, or "reentrant," oscillations are possible only if the total phase shift around the structure is an integral multiple of 2π radians. Thus, if there are N reentrant cavities in the anode structure, the phase shift between two adjacent cavities can be expressed as

$$\phi_n = \frac{2\pi m}{N} \tag{10-1-16}$$

where n is an integer indicating the nth mode of oscillation.

In order for oscillations to be produced in the structure, the anode dc voltage must be adjusted so that the average rotational velocity of the electrons corresponds to the phase velocity of the field in the slow-wave structure. Magnetron oscillators are ordinarily operated in the π mode. That is,

$$\phi_n = \pi \qquad (\pi \text{ mode}) \tag{10-1-17}$$

Figure 10-1-3 shows the lines of force in the π mode of an eight-cavity magnetron. It is evident that in the π mode the excitation is largely in the cavities, having opposite phase in successive cavities. The successive rise and fall of adjacent anode-cavity fields may be regarded as a traveling wave along the surface of the slow-wave structure. For the energy to be transferred from the moving electrons to the traveling field, the electrons must be decelerated by a retarding field when they pass through each anode cavity. If L is the mean separation between cavities, the phase constant of the fundamental-mode field is given by

$$\beta_0 = \frac{2\pi n}{NL} \tag{10-1-18}$$

The traveling-wave field of the slow-wave structure may be obtained by solving

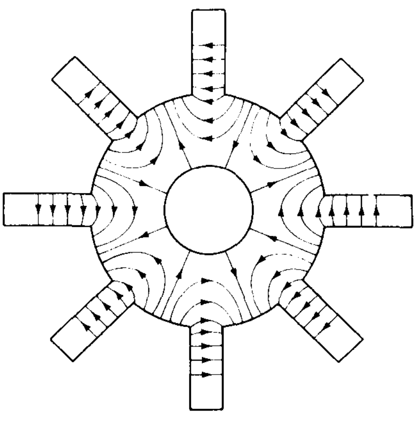


Figure 10-1-3 Lines of force in π mode of eight-cavity magnetron.

Maxwell's equations subject to the boundary conditions. The solution for the fundamental ϕ component of the electric field has the form [1]

$$E_{\phi 0} = jE_1 e^{j(\omega t - \beta_0 \phi)} \tag{10-1-19}$$

where E_1 is a constant and β_0 is given in Eq. (10-1-18). Thus, the traveling field of the fundamental mode travels around the structure with angular velocity

$$\frac{d\phi}{dt} = \frac{\omega}{\beta_0} \tag{10-1-20}$$

where $\frac{d\phi}{dt}$ can be found from Eq. (10-1-19).

When the cyclotron frequency of the electrons is equal to the angular frequency of the field, the interactions between the field and electron occurs and the energy is transferred. That is,

$$\omega_c = \beta_0 \frac{d\phi}{dt} \tag{10-1-21}$$

Power output and efficiency. The efficiency and power output of a magnetron depend on the resonant structure and the dc power supply. Figure 10-1-4 shows an equivalent circuit for a resonator of a magnetron.

where Y_e = electronic admittance
V = RF voltage across the vane tips
C = capacitance at the vane tips
L = inductance of the resonator
G_r = conductance of the resonator
G = load conductance per resonator

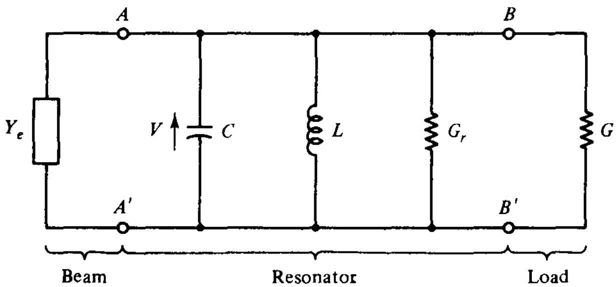


Figure 10-1-4 Equivalent circuit for one resonator of a magnetron.

Each resonator of the slow-wave structure is taken to comprise a separate resonant circuit as shown in Fig. 10-1-4. The unloaded quality factor of the resonator is given by

$$Q_{\text{un}} = \frac{\omega_0 C}{G_r} \tag{10-1-22}$$

where $\omega_0 = 2\pi f_0$ is the angular resonant frequency. The external quality factor of the load ciruit is

$$Q_{ex} = \frac{\omega_0 C}{G_\ell} \tag{10-1-23}$$

Then the loaded Q_ℓ of the resonant circuit is expressed by

$$Q_\ell = \frac{\omega_0 C}{G_r + G_\ell} \tag{10-1-24}$$

The circuit efficiency is defined as

$$\begin{aligned} \eta_c &= \frac{G_\ell}{G_\ell + G_r} \\ &= \frac{G_\ell}{G_{ex}} = \frac{1}{1 + Q_{ex}/Q_{un}} \end{aligned} \tag{10-1-25}$$

The maximum circuit efficiency is obtained when the magnetron is heavily loaded, that is, for $G_\ell \gg G_r$. Heavy loading, however, makes the tube quite sensitive to the load, which is undesirable in some cases. Therefore, the ratio of Q_ℓ/Q_{ex} is often chosen as a compromise between the conflicting requirements for high circuit efficiency and frequency stability.

The electronic efficiency is defined as

$$\eta_e = \frac{P_{gen}}{P_{dc}} = \frac{V_0 I_0 - P_{lost}}{V_0 I_0} \tag{10-1-26}$$

where P_{gen} = RF power induced into the anode circuit
P_{dc} = $V_0 I_0$ power from the dc power supply
V_0 = anode voltage
I_0 = anode current
P_{lost} = power lost in the anode circuit

The RF power generated by the electrons can be written as

$$\begin{aligned} P_{gen} &= V_0 I_0 - P_{lost} \\ &= V_0 I_0 - I_0 \frac{m}{2e} \frac{\omega_0^2}{\beta^2} + \frac{E_{max}^2}{B_z^2} \\ &= \frac{1}{2} N \, |V|^2 \frac{\omega_0 C}{Q_\ell} \end{aligned} \tag{10-1-27}$$

where N = total number of resonators
V = RF voltage across the resonator gap
E_{max} = $M_1 |V|/L$ is the maximum electric field
M_1 = $\sin\left(\beta_n \frac{\delta}{2}\right) \Big/ \left(\beta_n \frac{\delta}{2}\right) = 1$ for small δ is the gap factor for the π-mode operation
β = phase constant
B_z = magnetic flux density
L = center-to-center spacing of the vane tips

The power generated may be simplified to

$$P_{gen} = \frac{NL^2\omega_0 C}{2M_1^2 Q_\ell} E_{max}^2 \tag{10-1-28}$$

The electronic efficiency may be rewritten as

$$\eta_e = \frac{P_{gen}}{V_0 I_0} = \frac{1 - \dfrac{m\omega_0^2}{2eV_0\beta^2}}{1 + \dfrac{I_0 m M_1^2 Q_\ell}{B_z e NL^2 \omega_0 C}} \tag{10-1-29}$$

Example 10-1-1A: Pulsed Magnetron

An X-band pulsed conventional magnetron has the following operating parameters:

Anode voltage:	$V_0 = 5.5$ kV
Beam current:	$I_0 = 4.5$ A
Operating frequency:	$f = 9 \times 10^9$ Hz
Resonator conductance:	$G_r = 2 \times 10^{-4}$ mho
Loaded conductance:	$G_\ell = 2.5 \times 10^{-5}$ mho
Vane capacitance:	$C = 2.5$ pF
Duty cycle:	$DC = 0.002$
Power loss:	$P_{loss} = 18.50$ kW

Compute:

a. The angular resonant frequency
b. The unloaded quality factor
c. The loaded quality factor
d. The external quality factor
e. The circuit efficiency
f. The electronic efficiency

Solution

a. The angular resonant frequency is

$$\omega_r = 2 \times 9 \times 10^9 = 56.55 \times 10^9 \text{ rad}$$

b. The unloaded quality factor is

$$Q_{un} = \frac{56.55 \times 10^9 \times 2.5 \times 10^{-12}}{2 \times 10^{-4}} = 707$$

c. The loaded quality factor is

$$Q_l = \frac{56.55 \times 10^9 \times 2.5 \times 10^{-12}}{2 \times 10^{-4} + 2.5 \times 10^{-5}} = 628$$

d. The external quality factor is

$$Q_{ex} = \frac{56.55 \times 10^9 \times 2.5 \times 10^{-12}}{2.5 \times 10^{-5}} = 5655$$

e. The circuit efficiency is

$$\eta_c = \frac{1}{1 + 5655/707} = 11.11\%$$

f. The electronic efficiency is

$$\eta_e = \frac{5.5 \times 10^3 \times 4.5 - 18.5 \times 10^3}{5.5 \times 10^3 \times 4.5} = 25.25\%$$

State of the art. For many years, magnetrons have been the high-power sources in operating frequencies as high as 70 GHz. Military radar relies on conventional traveling-wave magnetrons to generate high-peak-power RF pulses. No other microwave devices can perform the same function with the same size, weight, voltage, and efficiency-range advantage as can the conventional magnetrons. At the present state of the art, a magnetron can deliver a peak power output of up to 40 MW with the dc voltage in the order of 50 kV at the frequency of 10 GHz. The average power outputs are up to 800 kW. Its efficiency is very high, ranging from 40 to 70%. Figure 10-1-5 shows the state of the art for U.S. high-power magnetrons.

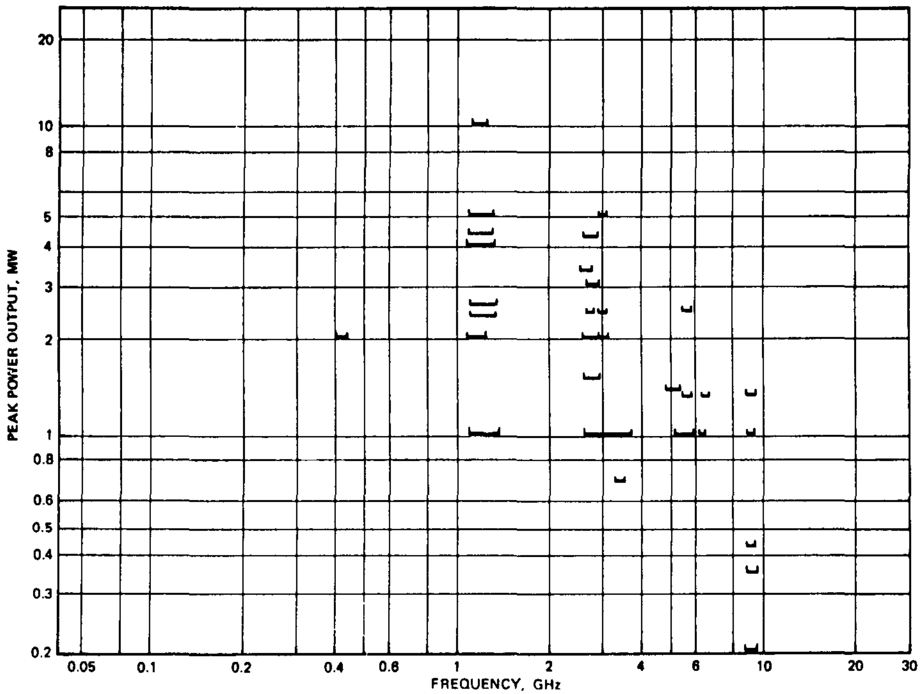


Figure 10-1-5 State of the art for U.S. high-power magnetrons.

Beacon magnetrons—miniature conventional magnetrons—deliver peak outputs as high as 3.5 kW, yet weigh less than two pounds. These devices are ideal for use where a very compact, low-voltage source of pulsed power is required, such as in airborne, missile, satellite, or Doppler systems. Most of the beacon magnetrons exhibit negligible frequency shift and provide long-life performance under the most severe environmental and temperature conditions.

The Litton L-5080 pulse magnetron, shown in Fig. 10-1-6, is a typical vane-strap magnetron oscillator. It has a maximum peak output power of 250 kW at a frequency range from 5.45 to 5.825 Ghz. Its duty cycle is 0.0012.

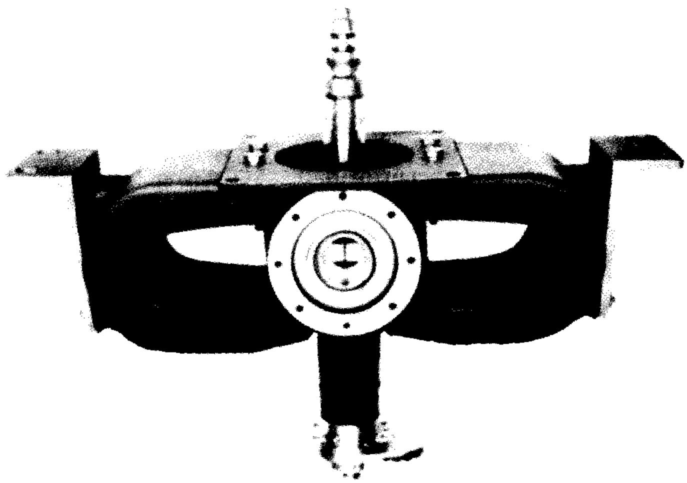


Figure 10-1-6 Photograph of Litton L-5080 magnetron. (*Courtesy of Litton Electron Tube Division.*)

10-1-2 Linear Magnetron

The schematic diagram of a linear magnetron is shown in Fig. 10-1-7. In the linear magnetron as shown in Fig. 10-1-7, the electric field E_x is assumed in the positive x direction and the magnetic flux density B_z in the positive z direction. The differential equations of motion of electrons in the crossed-electric and magnetic fields can be written from Eqs. (1-3-2a, b, c) as

$$\frac{d^2x}{dt^2} = -\frac{e}{m}\left(E_x + B_z\frac{dy}{dt}\right) \tag{10-1-30}$$

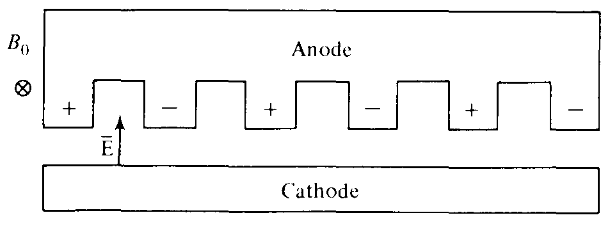


Figure 10-1-7 Schematic diagram of a linear magnetron.

$$\frac{d^2y}{dt^2} = \frac{e}{m}B_z\frac{dx}{dt} \tag{10-1-31}$$

$$\frac{d^2z}{dt^2} = 0 \tag{10-1-32}$$

where $\frac{e}{m} = 1.759 \times 10^{11}$ C/kg is the charge-to-mass of an electron
B_z = magnetic flux density in positive z direction
E_x = electric field in positive x direction

In general, the presence of space charges causes the field to be a nonlinear function of the distance x, and the complete solution of Eqs. (10-1-30) through (10-1-32) is not simple. Equation (10-1-31), however, can be integrated directly. Under the assumptions that the electrons emit from the cathode surface with zero initial velocity and that the origin is the cathode surface, Eq. (10-1-31) becomes

$$\frac{dy}{dt} = \frac{e}{m}B_z x \tag{10-1-33}$$

Equation (10-1-33) shows that, regardless of space charges, the electron velocity parallel to the electrode surface is proportional to the distance of the electron from the cathode and to the magnetic flux density B_z. How far the electron moves from the cathode depends on B_z and on the manner in which the potential V varies with x, which in turn depends on the space-charge distribution, anode potential, and electrode spacing.

If the space charge is assumed to be negligible, the cathode potential zero, and the anode potential V_0, the differential electric field becomes

$$\frac{dV}{dx} = \frac{V_0}{d} \tag{10-1-34}$$

where V_0 = anode potential in volts
d = distance between cathode and anode in meters

Substitution of Eq. (10-1-34) into Eq. (10-1-30) yields

$$\frac{d^2x}{dt^2} = \frac{e}{m}\left(\frac{V_0}{d} - B_z\frac{dy}{dt}\right) \tag{10-1-35}$$

Combination of Eqs. (10-1-33) and (10-1-35) results in

$$\frac{d^2x}{dt^2} + \left(\frac{e}{m}B_z\right)^2 x - \frac{e}{m}\frac{V_0}{d} = 0 \tag{10-1-36}$$

Solution of Eq. (10-1-36) and substitution of the solution into Eq. (10-1-33) yield the following equations for the path of an electron with zero velocity at cathode (origin point) as

$$x = \frac{V_0}{B_z\omega_c d}[1 - \cos(\omega_c t)] \tag{10-1-37}$$

$$y = \frac{V_0}{B_z \omega_c d}[\omega_c t - \sin(\omega_c t)] \tag{10-1-38}$$

$$z = 0 \tag{10-1-39}$$

where $\omega_c = \dfrac{e}{m} B_z$ is the cyclotron angular frequency

$f_c = 2.8 \times 10^6\, B_z$ is the cyclotron frequency in Hz

Equations (10-1-37) through (10-1-39) are those of a cycloid generated by a point on a circle of radius $V_0/(B_z\omega_c d)$ rolling on the plane of the cathode with angular frequency ω_c. The maximum distance to which the electron moves in a direction normal to the cathode is $2V_0 m/(B_z^2 ed)$. When this distance is just equal to the anode-cathode distnce d, the electrons graze the anode surface, and the anode current is cut off. Then the cutoff condition is

$$\frac{2V_0 m}{B_z^2 ed} = d \tag{10-1-40}$$

Let a constant K equal to

$$K = \frac{d^2 B_z^2}{V_0} = \frac{2m}{e} = 1.14 \times 10^{-11} \tag{10-1-41}$$

When the value of K is less than 1.14×10^{-11}, electrons strike the anode; when the value is larger than 1.14×10^{-11}, they return to the cathode. Figure 10-1-8 shows the electron path.

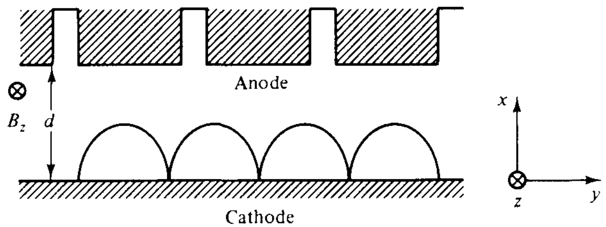


Figure 10-1-8 Electron path in a linear magnetron.

From Eq. (10-1-40), the Hull cutoff voltage for a linear magnetron is given by

$$V_{0c} = \frac{1}{2}\frac{e}{m} B_0^2 d^2 \tag{10-1-42}$$

where $B_0 = B_z$ is the magnetic flux density in the positive z direction. This means that if $V_0 < V_{0c}$ for a given B_0, the electrons will not reach the anode.

Similarly, the Hull cutoff magnetic flux density for a linear magnetron is expressed as

$$B_{0c} = \frac{1}{d}\sqrt{2\frac{m}{e} V_0} \tag{10-1-43}$$

This means that if $B_0 > B_{0c}$ for a given V_0, the electrons will not reach the anode.

Example 10-1-2: Linear Magnetron

A linear magnetron has the following operating parameters:

Anode voltage:	$V_0 = 10$ kV
Cathode current:	$I_0 = 1$ A
Magnetic flux density:	$B_0 = 0.01$ Wb/m^2
Distance between cathode and anode:	$d = 5$ cm

Compute:

a. The Hull cutoff voltage for a fixed B_0

b. The Hull cutoff mangetic flux density for a fixed V_0

Solution

a. The Hull cutoff voltage is

$$V_{0c} = \tfrac{1}{2} \times 1.759 \times 10^{11} \times (0.01)^2 \times (5 \times 10^{-2})^2$$
$$= 22.00 \text{ kV}$$

b. The Hull magnetic flux density is

$$B_{0c} = \frac{1}{5 \times 10^{-2}} \times \left(\frac{2 \times 10 \times 10^3}{1.759 \times 10^{11}}\right)^{1/2}$$
$$= 6.74 \text{ mWb/m}^2$$

Hartree condition. The Hull cutoff condition determines the anode voltage or magnetic field necessary to obtain nonzero anode current as a function of the magnetic field or anode voltage in the absence of an electromagnetic field. The Hartree condition can be derived as follows and as shown in Fig. 10-1-9.

The electron beam lies within a region extending a distance h from the cathode, where h is known as the hub thickness. The spacing between the cathode and anode is d. The electron motion is assumed to be in the positive y direction with a velocity

$$\mathcal{V}_y = -\frac{E_x}{B_0} = \frac{1}{B_0}\frac{dV}{dx} \tag{10-1-44}$$

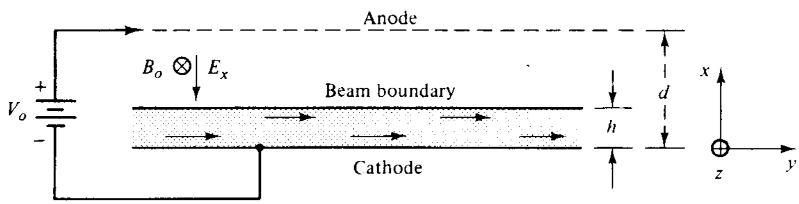


Figure 10-1-9 Linear model of a magnetron.

where $B_0 = B_z$ is the magnetic flux density in the positive z direction
V = potential

From the principle of energy conservation, we have

$$\tfrac{1}{2} m \mathcal{V}_y^2 = eV \tag{10-1-45}$$

Combining Eqs. (10-1-44) and (10-1-45) yields

$$\left(\frac{dV}{dx}\right)^2 = \frac{2eV}{m} B_0^2 \tag{10-1-46}$$

This differential equation may be rearranged as

$$\left(\frac{m}{2eB_0}\right)^{1/2} \frac{dV}{\sqrt{V}} = dx \tag{10-1-47}$$

Integration of Eq. (10-1-47) yields the potential within the electron beam as

$$V = \frac{eB_0^2}{2m} x^2 \tag{10-1-48}$$

where the constant of integration has been eliminated for $V = 0$ at $x = 0$. The potential and electric field at the hub surface are given by

$$V(h) = \frac{e}{2m} B_0^2 h^2 \tag{10-1-49}$$

and

$$E_x = -\frac{dV}{dx} = -\frac{\epsilon}{m} B_0^2 h \tag{10-1-50}$$

The potential at the anode is thus obtained from Eq. (10-1-50) as

$$\begin{aligned} V_0 &= -\int_0^d E_x \, dx \\ &= -\int_0^h E_x \, dx - \int_h^d E_x \, dx \\ &= V(h) + \frac{e}{m} B_0^2 h(d - h) \\ &= \frac{e}{m} B_0^2 h(d - h/2) \end{aligned} \tag{10-1-51}$$

The electron velocity at the hub surface is obtained from Eqs. (10-1-44) and (10-1-50) as

$$\mathcal{V}_y(h) = \frac{e}{m} B_0 h \tag{10-1-52}$$

For synchronism, this electron velocity is equal to the phase velocity of the slow-wave structure. That is,

$$\frac{\omega}{\beta} = \frac{e}{m}B_0h \tag{10-1-53}$$

For the π-mode operation, the anode potential is finally given by

$$V_{0h} = \frac{\omega B_0 d}{\beta} - \frac{m}{2e}\frac{\omega^2}{\beta^2} \tag{10-1-54}$$

This is the Hartree anode voltage equation that is a function of the magnetic flux density and the spacing beween the cathode and anode.

Example 10-1-2a: Linear Magnetron

A linear magnetron has the following operation parameters:

Anode voltage:	$V_0 = 15$ kV
Cathode current:	$I_0 = 1.2$ A
Operating frequency:	$f = 8$ GHz
Magnetric flux density:	$B_0 = 0.015$ Wb/m^2
Hub thickness:	$h = 2.77$ cm
Distance between anode and cathode:	$d = 5$ cm

Calculate:

a. The electron velocity at the hub surface
b. The phase velocity for synchronism
c. The Hartree anode voltage

Solution

a. The electron velocity is

$$\begin{aligned}\mathcal{V} &= 1.759 \times 10^{11} \times 0.015 \times 2.77 \times 10^{-2} \\ &= 0.73 \times 10^3 \text{ m/s}\end{aligned}$$

b. The phase velocity is

$$\mathcal{V}_{\text{ph}} = \frac{\omega}{\beta} = 0.73 \times 10^8 \text{ m/s}$$

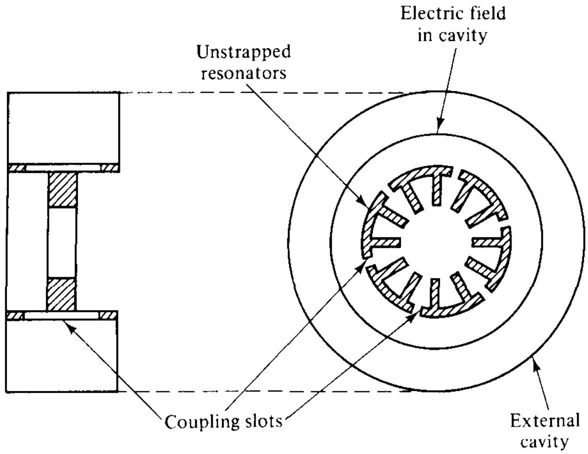
c. The Hartree anode voltage is

$$\begin{aligned}V_{0h} &= 0.73 \times 10^8 \times 0.015 \times 5 \times 10^{-2} - \frac{1}{2 \times 1.759 \times 10^{11}} \times (0.73 \times 10^8)^2 \\ &= 5.475 \times 10^4 - 1.515 \times 10^4 \\ &= 39.60 \text{ kV}\end{aligned}$$

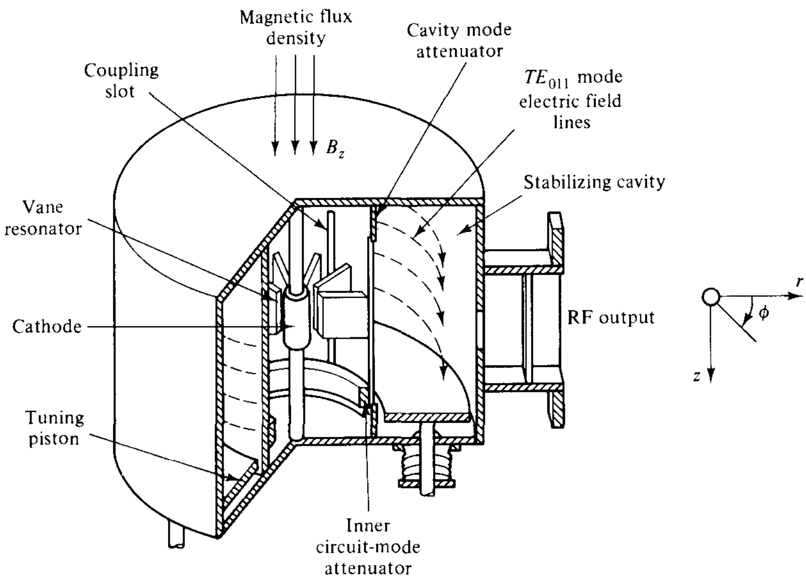
10-1-3 Coaxial Magnetron

The coaxial magnetron is composed of an anode resonator structure surrounded by an inner-single, high-Q cavity operating in the TE_{011} mode as shown in Fig. 10-1-10.

The slots in the back walls of alternate cavities of the anode resonator structure tightly couple the electric fields in these resonators to the surrounding cavity. In the



(a) Cross section



(b) Cutaway view

Figure 10-1-10 Schematic diagram of a coaxial magnetron. (*Courtesy of Varian Associates, Inc.*)

π-mode operation, the electric fields in every other cavity are in phase, and so they couple in the same direction into the surrounding cavity. As a result, the surrounding coaxial cavity stabilizes the magnetron in the desired π-mode oepration.

In the desired TE_{011} mode, the electric fields follow a circular path within the cavity and reduce to zero at the walls of the cavity. Current flow in the TE_{011} mode is in the walls of the cavity in circular paths about the axis of the tube. The undesired modes are damped out by the attenuator within the inner slotted cylinder near the ends of the coupling slots. The tuning mechanism is simple and reliable. As the straps are not required, the anode resonator for the coaxial magnetron can be larger and less complex than for the conventional strapped magnetron. Thus cathode loading is lower, and voltage gradients are reduced.

The Varian SFD-333TM magnetron, as shown in Fig. 10-1-11 is a typical X-band coaxial magnetron. It has a minimum peak power of 400 kW at a frequency range from 8.9 to 9.6 GHz. Its duty cycle is 0.0013. The nominal anode voltage is 32 kV, and the peak anode current is 32 A.

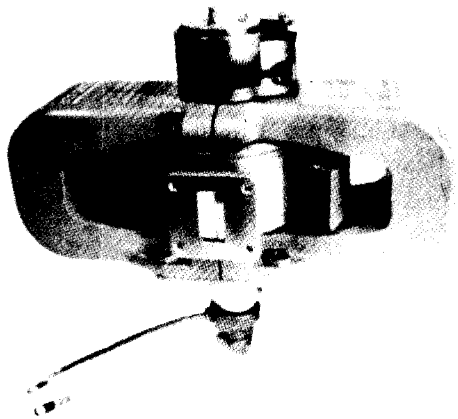


Figure 10-1-11 Photograph of Varian SFD-333TM coaxial magnetron (*Courtesy of Varian Associates, Inc.*)

10-1-4 Voltage-Tunable Magnetron

The voltage-tunable magnetron is a broadband oscillator with frequency changed by varying the applied voltage between the anode and sole. As shown in Fig. 10-1-12, the electric beam is emitted from a short cylindrical cathode at one end of the device. Electrons are formed into a hollow beam by the electric and magnetic forces near the cathode and then are accelerated radically outward from the cathode. The electron beam is then injected into the region between the sole and the anode. The beam rotates about the sole at the rate controlled by the axial magnetic field and the dc voltage applied between the anode and the sole.

The voltage-tunable magnetron uses a low-Q resonator, and its bandwidth may exceed 50% at low-power levels. In the π-mode operation, the bunch process of the hollow beam occurs in the resonator, and the frequency of oscillation is determined by the rotational velocity of the electron beam. In other words, the oscillation frequency can be controlled by varying the applied dc voltage between the anode and

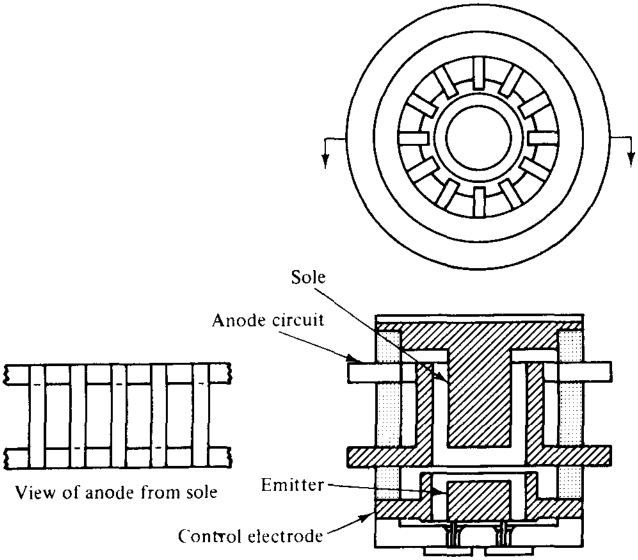


Figure 10-1-12 Cross-section view of a voltage-tunable magnetron. (*Courtesy of Varian Associates, Inc.*)

sole. Power output can be adjusted to some extent through the use of the control electrode in the electron gun. At high-power levels and high frequencies, the bandwidth percentage is limited. However, at low-power levels and low frequencies, the bandwidth may approach 70%.

10-1-5 Inverted Coaxial Magnetron

A magnetron can be built with the anode and cathode inverted—that is, with the cathode surrounding the anode. For some time, the basic problem of mode suppression prevented its use. In an inverted coaxial magnetron, the cavity is located inside a slotted cylinder, and a resonator vane array is arranged on the outside. The cathode is built as a ring around the anode. Figure 10-1-13 shows the schematic diagram of an inverted coaxial magnetron.

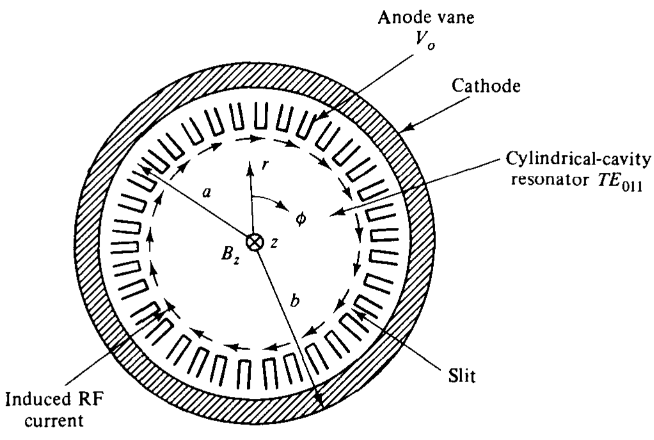


Figure 10-1-13 Schematic diagram of an inverted coaxial magnetron. (*Courtesy of Varian Associates, Inc.*)

Mathematically, the motion equations of the electrons in an inverted coaxial magnetron can be written from Eqs. (1-2-5a, b, c) as

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_z\frac{d\phi}{dt} \tag{10-1-55}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{d\phi}{dt} \tag{10-1-56}$$

where $\frac{e}{m} = 1.759 \times 10^{11}$ C/kg is the charge-to-mass ratio of electron

$B_0 = B_z$ is assumed in the positive z direction

Rearrangement of Eq. (10-1-56) results in the following form

$$\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z r\frac{dr}{dt} = \frac{1}{2}\omega_c\frac{d}{dt}(r^2) \tag{10-1-57}$$

where $\omega_c = \frac{e}{m}B_z$ is the cyclotron angular frequency. Integration of Eq. (10-1-57) yields

$$r^2\frac{d\phi}{dt} = \frac{1}{2}\omega_c r^2 + \text{constant} \tag{10-1-58}$$

at $r = b$, where b is the radius of the cathode cylinder, and $\frac{d\phi}{dt} = 0$, constant $= -\frac{1}{2}\omega_c b^2$. The angular velocity is expressed by

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{b^2}{r^2}\right) \tag{10-1-59}$$

Since the magnetic field does no work on the electrons, the kinetic energy of the electron is given by

$$\tfrac{1}{2}m\mathcal{V}^2 = eV \tag{10-1-60}$$

However, the electron velocity has r and ϕ components such as

$$\mathcal{V}^2 = \frac{2e}{m}V = \mathcal{V}_r^2 + \mathcal{V}_\phi^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 \tag{10-1-61}$$

at $r = a$, where a is the radius from the center of the cylinder to the edge of the anode, $V = V_0$, and $dr/dt = 0$.

When the electrons just graze the anode, Eqs. (10-1-60) and (10-1-61) become

$$\frac{d\phi}{dt} = \frac{1}{2}\omega_c\left(1 - \frac{b^2}{a^2}\right) \tag{10-1-62}$$

$$a^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m}V_0 \tag{10-1-63}$$

Substitution of Eq. (10-1-62) into Eq. (10-1-63) results in

$$a^2\left[\frac{1}{2}\omega_c\left(1-\frac{b^2}{a^2}\right)\right]^2=\frac{2e}{m}V_0 \tag{10-1-64}$$

The electron will acquire a tangential as well as a radial velocity. Whether the electron will just graze the anode and return back toward the cathode depends on the relative magnitudes of the anode voltage V_0 and the magnetic flux density B_0. The cutoff condition can be obtained from Eq. (10-1-64) as

$$V_{0c}=\frac{e}{8m}B_0^2a^2\left(1-\frac{b^2}{a^2}\right)^2 \tag{10-1-65}$$

This means that if $V_0 < V_{0c}$ for a given B_0, the electrons will not reach the anode. Equation (10-1-65) is often called the *Hull cutoff voltage equation*. Similarly, the magnetic cutoff condition is expressed by

$$B_{0c}=\frac{-\left(8V_0\dfrac{m}{e}\right)^{1/2}}{a\left(1-\dfrac{b^2}{a^2}\right)} \tag{10-1-66}$$

This means that if $B_0 > B_{0c}$ for a given V_0, the electrons will not reach the anode. Equation (10-1-66) is called the *Hull cutoff magnetic equation*.

The advantage of an inverted coaxial magnetron design is that the cathode current density can be reduced to one-tenth of that used in cathode-centered magnetrons. Thus the millimeter magnetron is a practical and long-life device. The output waveguide can be in the circular electric mode that has extremely low transmission loss. Figure 10-1-14 compares the inverted coaxial magnetron with a conventional magnetron designed for the same frequency. It should be noted that the cathode sizes are quite different.

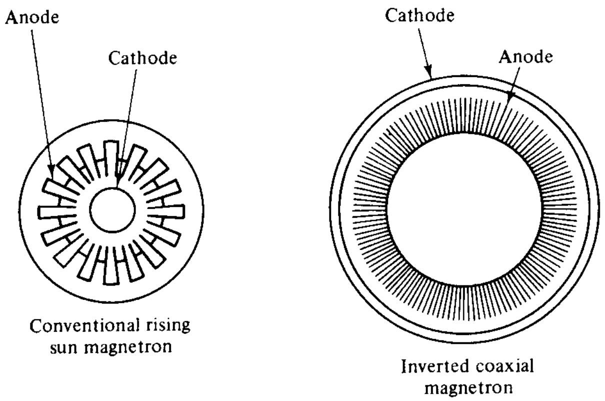


Figure 10-1-14 Comparison of a conventional magnetron with an inverted coaxial magnetron. (*Courtesy of Varian Associates, Inc.*)

Example 10-1-5: Inverted Coaxial Magnetron

An inverted coaxial magnetron has the following parameters:

Anode voltage:	$V_0 = 10$ kV
Cathode current:	$I_0 = 2$ A
Anode radius:	$a = 3$ cm
Cathode radius:	$b = 4$ cm
Magnetic flux density:	$B_0 = 0.01$ Wb/m^2

Determine:

a. The cutoff voltage for a fixed B_0

b. The cutoff magnetic flux density for a fixed V_0

Solution

a. The cutoff voltage is

$$V_{0c} = \frac{1}{8} \times 1.759 \times 10^{11} \times (0.01)^2 \times (3 \times 10^{-2})^2 \times \left(1 - \frac{4^2}{3^2}\right)^2 = 1.20 \text{ kV}$$

b. The cutoff magnetic flux density is

$$B_{0c} = -\left(\frac{8 \times 10 \times 10^3}{1.759 \times 10^{11}}\right)^{1/2} \left[3 \times 10^{-2}\left(1 - \frac{4^2}{3^2}\right)\right]^{-1} = 0.0289 \text{ Wb/m}^2$$

10-1-6 Frequency-Agile Coaxial Magnetron

A frequency-agile coaxial magnetron differs from a standard tunable magnetron. The frequency agility (FA) of a coaxial magnetron is defined as the capability to tune the output frequency of the radar with sufficiently high speed to produce a pulse-to-pulse frequency change greater than the amount required effectively to obtain decorrelation of adjacent radar echoes. The frequency-agile magnetron, together with appropriate receiver integration circuits, can reduce target scintillation, increase the detectability of target in a clutter environment, and improve resistance to electronic countermeasures (ECM). The increase of the pulse-to-pulse frequency separation will improve the radar system performance. Furthermore, the greater the pulse-to-pulse frequency separation, the more difficult it will be to center a jamming transmitter on the radar frequency for effective interference with system operation.

The frequency-agile coaxial magnetrons are classified into three types:

1. *Dither magnetrons:* The output RF frequency varies periodically with a constant excursion, constant rate, and a fixed center frequency.
2. *Tunable/dither magnetrons:* The output RF frequency varies periodically with a constant excursion and constant rate, but the center frequency can be manually tuned by hand or mechanically tuned by a servomotor.
3. *Accutune magnetrons:* The output RF frequency variations are determined by the waveforms of an externally generated, low-level voltage signal. With proper selection of a tuning waveform, the accutune magnetron combines with features of dither and tunable/dither magnetrons, together with a capability for varying the excursion, rate, and tuning waveform. Figure 10-1-15 shows an accutune magnetron.

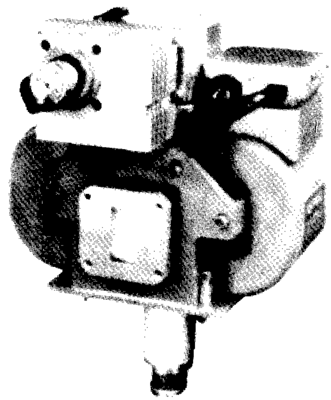


Figure 10-1-15 X-band accutune magnetron VMX-1430. (*Courtesy of Varian Associates, Inc.*)

The X-band frequency-agile coaxial magnetron VMX-1430 is a typical agile magnetron. Its pulse voltage is 15 kV, and pulse current is 15 A. Its maximum duty cycle is 0.0011, and accutune range is 1 GHz. Its center frequency is 9.10 GHz, and peak output power is 90 kW. The agile rate and agile excursion are shown in Fig. 10-1-16.

From Fig. 10-1-16, it can be seen that the agile rate is the number of times per second that the transmitter frequency traverses the agile excursion and returns to its starting frequency. Similarly, the agile excursion is defined as the total frequency variation of the transmitter during agile operation.

The number of pulses that can be effectively integrated cannot be greater than the number of pulses placed on the target during one scan of the antenna. Therefore, the antenna beamwidth and scan rate become factors that must also be considered in determining the integration period of the radar. Consequently, a design value for agile excursion can now be expressed in terms of radar operating parameters as

$$\text{Agile excursion} = \frac{N}{\tau} \qquad (10\text{-}1\text{-}67)$$

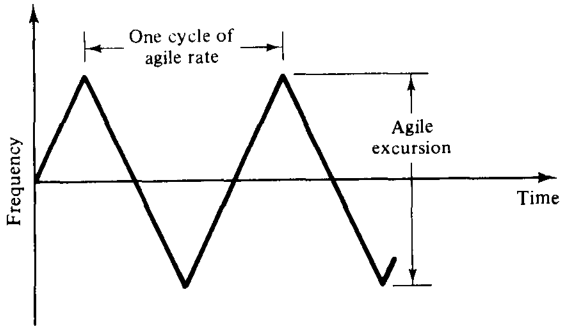


Figure 10-1-16 Agile rate and agile excursion.

where N = number of pulses placed on the target during one radar scan, say, 20, whichever is smaller

τ = shortest pulse duration used in the system

The frequency, or *pulse repetition rate (PRR)*, is given by

$$f = \frac{\text{DC}}{\tau} \tag{10-1-68}$$

where DC = duty cycle is the ratio of the pulse duration over the repetition period for a pulse. The duty cycle is defined as

$$\text{Duty cycle} = \frac{\text{Pulse duration}}{\text{Pulse repetition period}} = \frac{\tau}{T} = \tau f \tag{10-1-69}$$

Hence, the agile rate can be written as

$$\text{Agile rate} = \frac{1}{2T} \tag{10-1-70}$$

where the 2 in the denominator is counted for the fact that two excursions through the agile frequency range occur during each cycle of agile rate.

Example 10-1-6: Frequency-Agile Magnetron

A frequency-agile coaxial magnetron has the following operating parameters:

Pulse duration:	τ = 0.20, 0.40, 0.80 μs
Duty cycle:	DC = 0.001
Pulse rate on target:	N = 14 per scan

Determine:

a. The agile excursion
b. The pulse-to-pulse frequency separation
c. The signal frequency
d. The time for N pulses
e. The agile rate

Solution

a. The agile excursion is

$$\text{Agile excursion} = \frac{14}{0.2 \times 10^{-6}} = 70 \text{ MHz}$$

b. The pulse-to-pulse frequency separation is

$$f_p = \frac{1}{\tau} = \frac{1}{0.20 \times 10^{-6}} = 5 \text{ MHz}$$

c. The signal frequency is

$$f = \frac{\text{DC}}{\tau} = \frac{0.001}{0.20 \times 10^{-6}} = 5 \text{ kHz}$$

d. The time for 14 pulses per second is

$$\text{Time} = \frac{\text{N}}{f} = \frac{14}{5000} = 2.8 \text{ ms}$$

e. The agile rate is

$$\text{Agile rate} = \frac{1}{2 \times 0.0028} = 178.57 \text{ Hz}$$

10-2 FORWARD-WAVE CROSSED-FIELD AMPLIFIER (FWCFA OR CFA)

The crossed-field amplifier (CFA) is an outgrowth of the magnetron. CFAs can be grouped by their mode of operation as forward-wave or backward-wave types and by their electron stream source as emitting-sole or injected-beam types. The first group concerns the direction of the phase and group velocity of the energy on the microwave circuit. This can be seen from the ω-β diagram in Fig. 9-5-5 as discussed in Sec. 9-5. Since the electron stream reacts to the RF electric field forces, the behavior of the phase velocity with frequency is of prime concern. The second group emphasizes the method by which electrons reach the interaction region and how they are controlled. This can be seen in the schematic diagrams of Fig. 10-2-1.

In the forward-wave mode, the helix-type slow-wave structure is often selected as the microwave circuit for the crossed-field amplifier; in the backward-wave mode, the strapped bar line represents a satisfactory choice. A structure of strapped crossed-field amplifier is shown in Fig. 10-2-2.

10-2-1 Principles of Operation

In the emitting-sole tube, the current emanated from the cathode is in response to the electric field forces in the space between the cathode and anode. The amount of

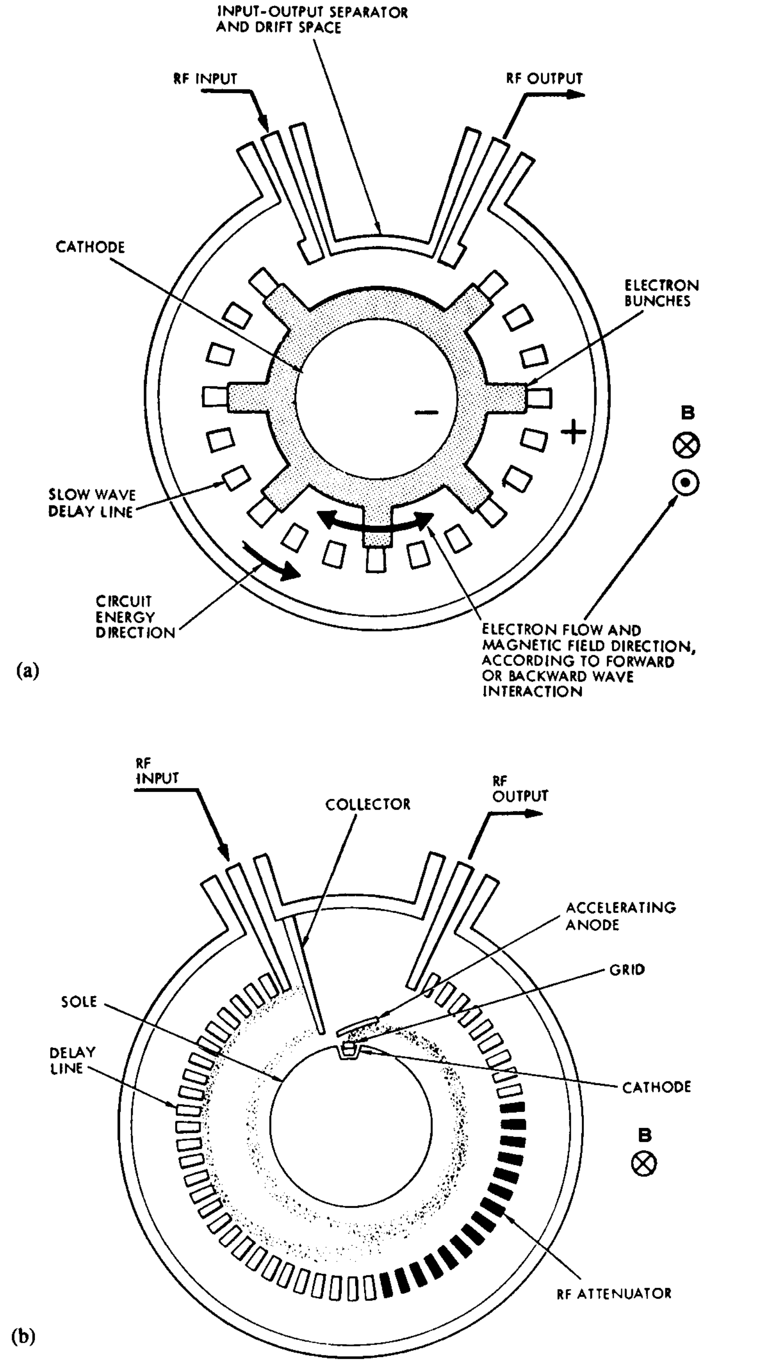


Figure 10-2-1 Schematic diagrams of CFAs. (*After J. F. Skowron [2], reprinted by permission of IEEE, Inc.*)

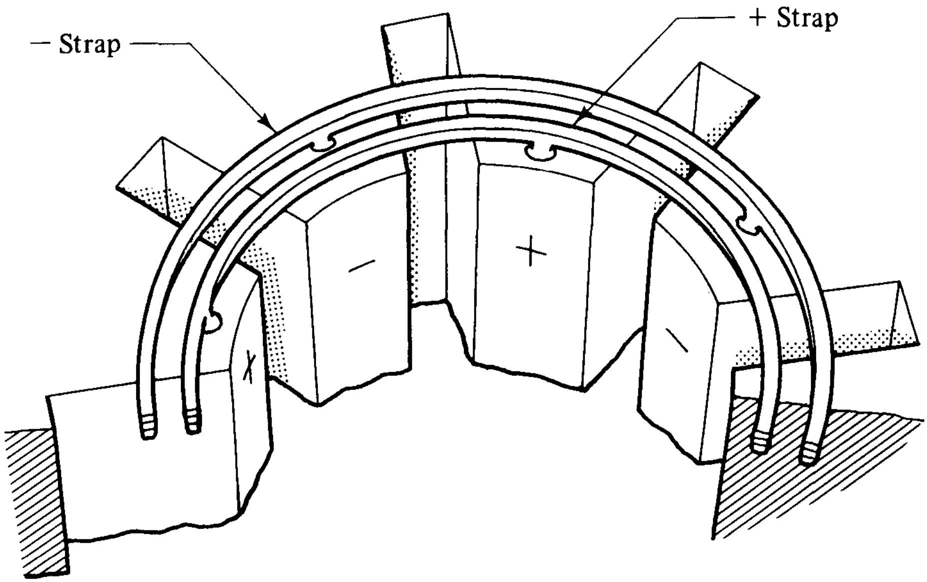


Figure 10-2-2 Diagram of a strapped CFA. (*After J. F. Skowron [2], reprinted by permission of IEEE, Inc.*)

current is a function of the dimension, the applied voltage, and the emission properties of the cathode. The perveance of the interaction geometry tends to be quite high, about 5 to 10 $\times$ 10^{-10}, which results in a high-current and high-power capability at relative low voltage. In the injected-beam tube the electron beam is produced in a separate gun assembly and is injected into the interaction region.

The beam-circuit interaction features are similar in both the emitting-sole and the injected-beam tubes. Favorably phased electrons continue toward the positively polarized anode and are ultimately collected, whereas unfavorably phased electrons are directed toward the negative polarized electrode.

In linear-beam interaction, as discussed for traveling-wave tubes in Sec. 9-5, the electron stream is first accelerated by an electric gun to the full dc velocity; the dc velocity is approximately equal to the axial phase velocity of the RF field on the slow-wave structure. After interaction occurs, the spent electron beam leaves the interaction region with a low-average velocity. The difference in velocity is accounted for by the RF energy created on the microwave circuit. In the CFA, the electron is exposed to the dc electric field force, magnetic field force, and the electric field force of the RF field, and even to the space-charge force from other electrons. The last force is normally not considered in analytic approaches because of its complexity. Under the influence of the three forces, the electrons travel in spiral trajectories in a direction tending along equipotentials. The exact motion has been subject to much analysis by means of a computer. Figure 10-2-3 shows the pattern of the electron flow in the CFA by computerized techniques [2]. It can be seen that when the spoke is positively polarized or the RF field is in the positive half cycle, the electron speeds up toward the anode; while the spoke is negatively polarized or the RF field is

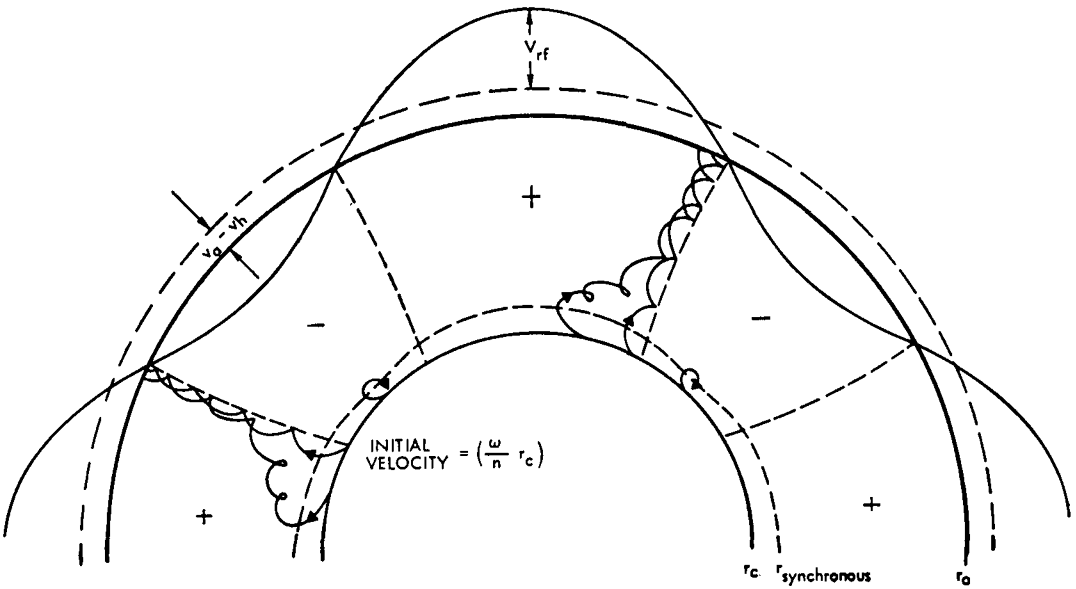


Figure 10-2-3 Motion of electrons in CFA. (*After J. F. Skowron [2], reprinted by permission of IEEE, Inc.*)

in the negative half cycle, the electrons are returned toward the cathode. Consequently, the electron beam moves in a spiral path in the interaction region.

The total power generated in a given CFA is independent of the RF input power, as long as the input power exceeds the threshold value for spoke stability at the input. The power generated can be increased only by increasing the anode voltage and current. Neglecting circuit attenuation, the output power of the CFA is equal to the sum of the input power and the power generated in the interaction region. That is, the power gain of a CFA is given by

$$g = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{in}} + P_{\text{gen}}}{P_{\text{in}}} = 1 + \frac{P_{\text{gen}}}{P_{\text{in}}} \tag{10-2-1}$$

where $P_{\text{out}} = P_{\text{in}} + P_{\text{gen}}$

P_{in} = RF input power

P_{gen} = RF power induced into the anode circuit by electrons

Therefore, the CFA is not a linear amplifier but rather is termed a *saturated amplifier*.

The efficiency of a CFA is defined as the product of the electronic efficiency η_e and the circuit efficiency η_c. The electronic efficiency η_e is defined as in Eq. 10-1-29). The overall efficiency is then expressed as

$$\eta = \eta_c \eta_e = \frac{P_{out} - P_{in}}{V_{a0} I_{a0}} \tag{10-2-2}$$

where P_{out} = RF output power
P_{in} = RF input power
$\eta_e = P_{gen}/P_{dc}$
$P_{dc} = V_{a0} I_{a0}$ is dc power
V_{a0} = anode dc voltage
I_{a0} = anode dc current

The circuit efficiency is defined as

$$\eta_c = \frac{\eta}{\eta_e} = \frac{P_{out} - P_{in}}{P_{gen}} \tag{10-2-3}$$

where $P_{gen} = \eta_e V_{a0} I_{a0}$

Since the power generated per unit length is constant, the output power is given by

$$\begin{aligned} P_{out} &= P_{in} e^{-2\alpha\ell} + \int_0^{\ell} \frac{P_{gen}}{\ell} e^{-2\alpha(\ell-\phi)}\, d\phi \\ &= P_{in} e^{-2\alpha\ell} + \frac{P_{gen}}{2\alpha\ell}(1 - e^{-2\alpha\ell}) \end{aligned} \tag{10-2-4}$$

where α = circuit attenuation constant
ℓ = circuit length in ϕ direction

Substitution of Eq. (10-2-4) in Eq. (10-2-3) results in the circuit-efficiency equation as

$$\eta_c = \left(\frac{1}{2\alpha\ell} - \frac{P_{in}}{P_{gen}}\right)(1 - e^{-2\alpha\ell}) \tag{10-2-5}$$

The term P_{in}/P_{gen} becomes negligible for high-gain CFA.

Assume that the input signal is sufficiently strong for spoke stability, that the RF power grows linearly with distance along the circuit, that the dc current for the spoke is constant, and that the back-bombardment loss is not considered, then the electronic-efficiency equation can be derived as follows. The average drift electron velocity at any position is given by

$$\mathcal{V} = \frac{E}{B} \tag{10-2-6}$$

where E = totall electric field at the position under consideration
B = magnetic flux density at the position

The power flow at any position is related to the RF field and the beam-coupling impedance of the circuit by

$$P = \frac{E_{max}^2}{2\beta^2 Z_c} \tag{10-2-7}$$

where E_{max} = peak electric field
$\beta = \omega/\mathcal{V}$ is phase constant
Z_c = beam-coupling impedance

The power loss per spoke due to the electron motion toward the anode at any position is derived from Eq. (10-2-6) as

$$P_s = V_{s0}I_{s0} = I_{s0}\frac{m}{e}\frac{\beta^2 Z_c P}{B^2} \tag{10-2-8}$$

where V_{s0} =dc voltage per spoke
I_{s0} =dc current per spoke

Since the power varies linearly with position, the average power loss over the entire circuit length is

$$P_{s,\text{avg}} = I_{s0}\frac{m}{2e}\frac{\beta^2 Z_c}{B^2}(P_{\text{in}} + P_{\text{out}}) \tag{10-2-9}$$

By using Eqs. (10-2-1), (10-2-9), and $\mathcal{V}\beta = \omega$, the total power loss for all the spokes is given by

$$P_{\text{lost}} = I_{a0}\frac{m}{2e}\left(\frac{\omega}{\beta}\right)^2 + I_{a0}\frac{m}{2e}\frac{\beta^2 Z_c}{B^2}\left(\frac{g+1}{g-1}\right)P_{\text{gen}} \tag{10-2-10}$$

where g = power gain.

Finally, the electronic efficiency is given by

$$\eta_e = \frac{P_{\text{gen}}}{V_{a0}I_{a0}} = \frac{1 - \dfrac{m\omega^2}{2eV_{a0}\beta^2}}{1 + \dfrac{I_{a0}}{B^2}\dfrac{m\beta^2 Z_c}{2e}\left(\dfrac{g+1}{g-1}\right)} \tag{10-2-11}$$

10-2-2 Microwave Characteristics

The crossed-field amplifier (CFA) is characterized by its low or moderate power gain, moderate bandwidth, high efficiency, saturated amplification, small size, low weight, and high perveance. These features have allowed the CFA to be used in a variety of electronic systems ranging from low-power and high-reliability space communications to multimegawatt, high average power, coherent pulsed radar. Figure 10-2-4 shows the present state of the art for the U.S. high-power forward-wave crossed-field amplifiers.

The Raytheon QKS-1541 amplifier, shown in Fig. 10-2-5, is a typical forward-wave CFA with an average power of 14 kW at frequency range from 2.9 to 3.1 GHz. Its pulse width is 28 s. Its typical peak anode voltage is 44 to 56 kV and peak anode current is 58 A.

Example 10-2-1: Crossed-Field Amplifier

A CFA operates under the following parameters:

Anode dc voltage:	$V_{a0} = 2$ kV
Anode dc current:	$I_{a0} = 1.5$ A

Electronic efficiency: $\eta_e = 20\%$

RF input power: $P_{in} = 80$ W

Calculate:

a. The induced RF power

b. The total RF output power

c. The power gain in decibels

Solution

a. The induced RF power is

$$P_{gen} = 0.20 \times 2 \times 10^3 \times 1.5 = 600 \text{ W}$$

b. The RF output power is

$$P_{out} = P_{in} - P_{gen} = 80 + 600 = 680 \text{ W}$$

c. The power gain is

$$g = \frac{P_{out}}{P_{in}} = \frac{680}{80} = 8.50 = 9.3 \text{ dB}$$

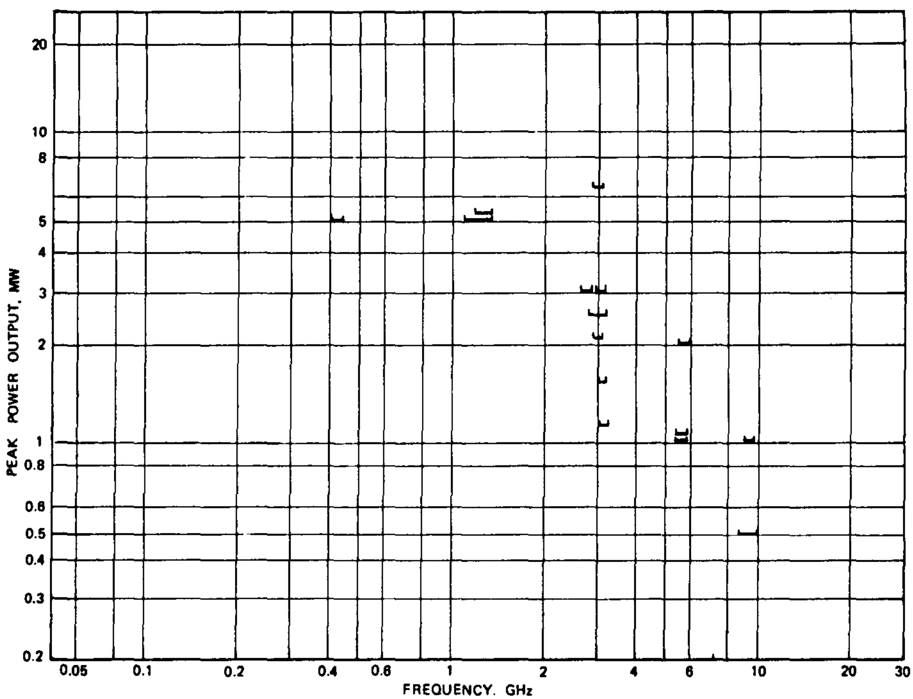


Figure 10-2-4 State of the art for U.S. high-power CFAs.

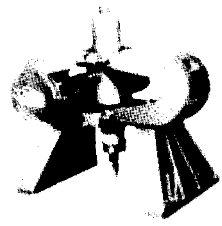


Figure 10-2-5 Photograph of Raytheon QKS-1541 CFA *(Courtesy of Raytheon Company, Microwave Tube Operation.)*

10-3 BACKWARD-WAVE CROSSED-FIELD AMPLIFIER (AMPLITRON)

The trade name of the backward-wave crossed-field amplifier (BWCFA) is Amplitron, and its schematic diagram is shown in Fig. 10-3-1.

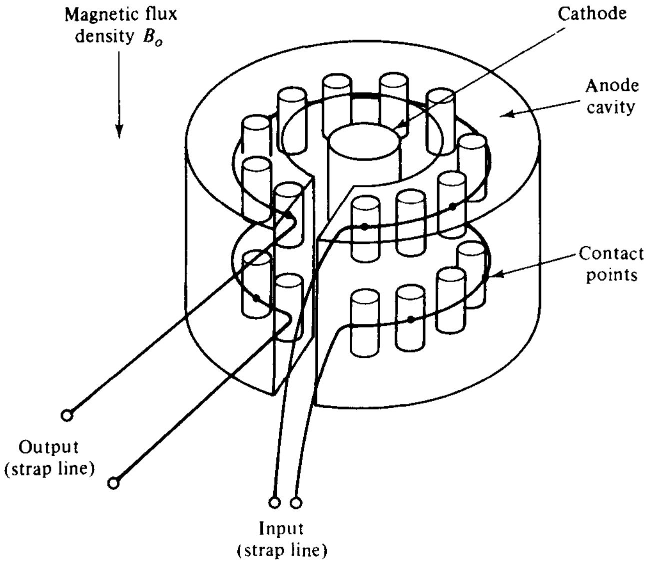


Figure 10-3-1 Schematic diagram of Amplitron.

The anode cavity and pins comprise the resonator circuits. A pair of pins and the cavity are excited in opposite phase by the strap line. The electron beam and the electromagnetic waves interact in the resonant circuits. The BWCFA can deliver 3-MW pulse with 10-μs duration at S-band, and the tube gain reaches 8 dB.

The highly successful QK434 Amplitron produced by Raytheon exhibited stable gain as high as 16 dB, power output levels ranging from a few hundred kilowatts to 3 MW, and efficiencies ranging from 60% for normal power levels to 76% for high-power, low-gain operation. The tube is commonly used in air surveillance radar and military pulsed radar. Figure 10-3-2 pictures the QK434 Amplitron.

The two-stage superpower Amplitron, also manufactured by Raytheon, generated 425 kW of CW power at an efficiency of 76%. The gain was 9 dB and the band-

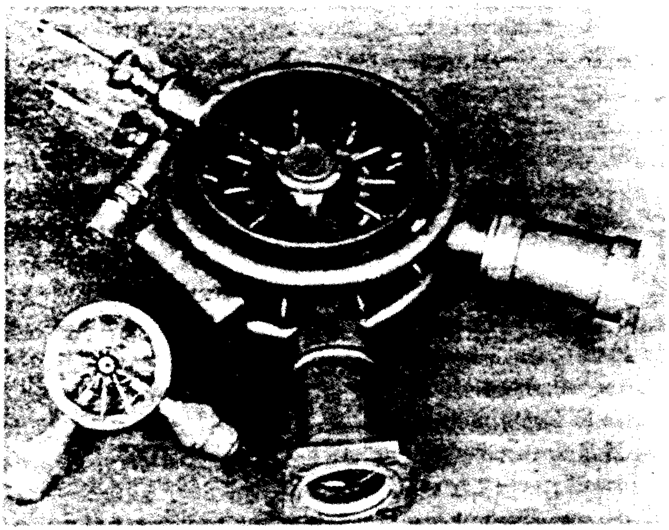


Figure 10-3-2 Photograph of QK-434 Amplitron. (*From W. C. Brown [3]; reprinted by permission of IEEE, Inc.*)

width was 5% at a mean frequency of 3 GHz. The tube was used for all high-data-rate transmission in the Apollo program. Figure 10-3-3 shows the two-stage superpower Amplitron.

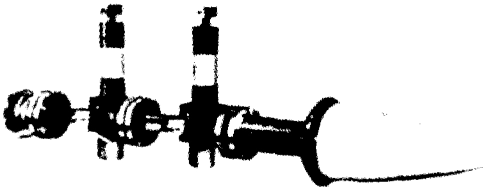


Figure 10-3-3 Photograph of a two-stage superpower Amplitron. (*From W. C. Brown [3]; reprinted by permission of IEEE, Inc.*)

The circuit and electronic equations for the *M*-type amplifier and oscillator were developed by several authors [4, 5]. The basic secular equation including the space-charge effect is given by

$$(\gamma^2 - \gamma_0^2)(j\beta_e - \gamma)\,[(j\beta_e - \gamma)^2 + \beta_m^2]$$
$$= -j\beta_e\gamma_0\gamma^2\left[(j\beta_e - \gamma) + j\frac{2\alpha}{1+\alpha^2}B_m\right]H^2 \qquad (10\text{-}3\text{-}1)$$

where γ_0 = circuit propagation constant

γ = harmonic wave propagation constant

$\beta_e = \dfrac{\omega}{\mathcal{V}_0}$ is the electron-beam phase constant

$\mathcal{V}_0 = \sqrt{\dfrac{2e}{m}V_0}$ is the dc electron-beam velocity

$$\beta_m = \frac{\omega_c}{\mathcal{V}_0} = \frac{e}{\mathcal{V}_0 m} B_0 \text{ is the cyclotron phase constant}$$

$$\omega_c = \frac{e}{m} B_0 \text{ is the cyclotron angular frequency}$$

$$B_0 = \text{crossed-magnetic flux density}$$

$$H_2 = 2(1 + \alpha^2)\phi^2 C^3$$

$$C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3} \text{ is the gain parameter}$$

$$\phi = A \exp(-j\gamma y) + B \exp(j\gamma y) \text{ is the wave equation}$$

$$\alpha = \frac{A \exp(-j\gamma y) - B \exp(j\gamma y)}{A \exp(-j\gamma y) + B \exp(j\gamma y)} = j \frac{1}{\gamma\phi} \frac{d\phi}{dy} \text{ is a factor}$$

In general, there are five solutions of γ from Eq. (10-3-1). Let

$$\gamma_0 = j\beta \tag{10-3-2}$$

$$\gamma = j\beta(1 + p) \tag{10-3-3}$$

where p is a very small constant ($p \ll 1$).

Substituting Eqs. (10-3-2) and (10-3-3) in Eq. (10-3-1) we have

$$p\left(\frac{\beta_e}{\beta} - 1 - p\right)\left[\left(\frac{\beta_e}{\beta} - 1 - p\right)^2 - \left(\frac{\beta_m}{\beta}\right)^2\right]$$
$$= \frac{-B_e}{2\beta}\left[\left(\frac{\beta_e}{\beta} - 1 - P\right) + \frac{2\alpha\beta_m}{(1 + \alpha^2)\beta}\right]H^2 \tag{10-3-4}$$

The right-hand term is small, as it is not much different from C, so that if either one of the left-hand terms is also small, Eq. (10-3-4) will be satisfied.

If $\beta_e \doteq \beta$ is assumed, the first factor is small and the solution for p becomes

$$p = \pm j\left(\frac{\alpha}{1 + \alpha^2}\right)^{1/2}\left(\frac{\beta}{\beta_m}\right)^{1/2} H \tag{10-3-5}$$

If $\beta_e \pm \beta_m \doteq \beta$ is assumed, the second factor is small and the solution for p is given by

$$p^2 = \pm\frac{1}{4}\frac{(1 \pm \alpha)^2}{1 + \alpha^2}\left(\frac{\beta}{\beta_m} \pm 1\right)H^2 \tag{10-3-6}$$

From the definition of p, gain is only obtained when p is imaginary. From Eq. (10-3-5), this only happens when α is positive. In Eq. (10-3-6), p is imaginary when $\beta_e + \beta_m = \beta$ and α is less than unity. This last condition comes from rewriting Eq. (10-3-6) as

$$p = \pm\frac{1}{2}\frac{1 - \alpha}{(1 + \alpha)^{1/2}}\left(\frac{\beta_e}{\beta_m}\right)^{1/2} H \tag{10-3-7}$$

The solution for an M-type backward-wave amplifier can be obtained by setting

$$\gamma = -j\beta(1 + p) \tag{10-3-8}$$

The only possible solution for $\beta + \beta_e = \beta_m$ is

$$p = \pm j\frac{1}{2}\frac{1 + \alpha}{(1 + \alpha^2)^{1/2}}\left(\frac{\beta_e}{\beta_m}\right)^{1/2} H \tag{10-3-9}$$

This gives rise to increasing backward waves for all values of α except $\alpha = -1$. Because $H \propto C^{3/2}$, Eqs. (10-3-5), (10-3-7), and (10-3-9) show that the gain per unit length in M-type devices is lower than in O-type devices using similar circuits because $C^{3/2} < C$.

Example 10-3-1: Amplitron Characteristics

An Amplitron has the following operating parameters:

Anode voltage:	$V_0 = 15$ kV
Anode current:	$I_0 = 3$ A
Magnetic flux density:	$B_0 = 0.2$ Wb/m^2
Operating frequency:	$f = 8$ GHz
Characteristic impedance:	$Z_0 = 50\ \Omega$

Determine:

a. The dc electron-beam velocity
b. The electron-beam phase constant
c. The cyclotron angular frequency
d. The cyclotron phase constant
e. The gain parameter

Solution

a. The dc electron-beam velocity is

$$\mathcal{V}_0 = 0.593 \times 10^6 \times (15 \times 10^3)^{1/2} = 0.762 \times 10^8 \text{ m/s}$$

b. The electron-beam phase constant is

$$\beta_e = \frac{\omega}{\mathcal{V}_0} = \frac{2\pi \times 8 \times 10^9}{0.762 \times 10^5} = 692.36 \text{ rad/m}$$

c. The cyclotron angular frequency is

$$\omega_c = \frac{e}{m}B_0 = 1.759 \times 10^{11} \times 0.2 = 35.18 \times 10^9 \text{ rad/s}$$

d. The cyclotron phase constant is

$$\beta_m = \frac{\omega_c}{\mathcal{V}_0} = \frac{35.18 \times 10^9}{0.726 \times 10^3} = 484.57 \qquad \text{rad/m}$$

e. The gain parameter is

$$C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3} = \left(\frac{3 \times 50}{4 \times 15 \times 10^3}\right)^{1/3} = 0.136$$

10-4 BACKWARD-WAVE CROSSED-FIELD OSCILLATOR (CARCINOTRON)

The backward-wave crossed-field oscillator of M-Carcinotron has two configurations: linear M-carcinotron and circular M-carcinotron.

Linear *M*-Carcinotron. The M-Carcinotron oscillator is an M-type backward-wave oscillator. The interaction between the electrons and the slow-wave structure takes place in a space of crossed field. A linear model of the M-Carcinotron oscillator is shown in Fig. 10-4-1.

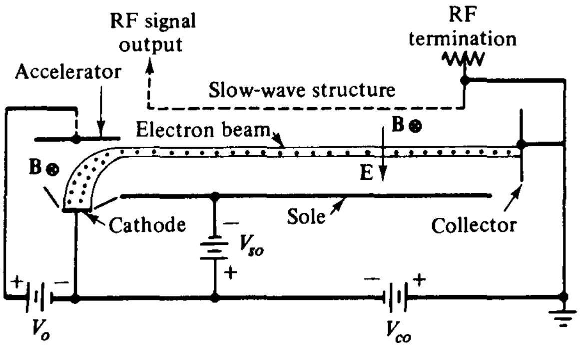


Figure 10-4-1 Linear model of an M-Carcinotron oscillator. (*From J. V. Gewartowski and H. A. Watson [6]; reprinted by permission of Van Nostrand Company.*)

The slow-wave structure is in parallel with an electrode known as the sole. A dc electric field is maintained between the grounded slow-wave structure and the negative sole. A dc magnetic field is directed into the page. The electrons emitted from the cathode are bent through a 90° angle by the magnetic field. The electrons interact with a backward-wave space harmonic of the circuit, and the energy in the circuit flows opposite to the direction of the electron motion. The slow-wave structure is terminated at the collector end, and the RF signal output is removed at the electron-gun end. Since the M-Carcinotron is a crossed-field device, its efficiency is very high, ranging from 30 to 60%.

The perturbed electrons moving in synchronism with the wave in a linear M-Carcinotron are shown in Fig. 10-4-2.

Electrons at position A near the beginning of the circuit are moving toward the circuit, whereas electrons at position B are moving toward the sole. Farther down

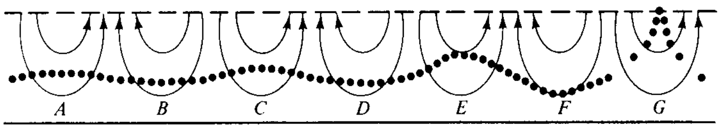


Figure 10-4-2 Beam electrons and electric field lines in an M-Carcinotron.

the circuit, electrons at position *C* are closer to the circuit, and electrons at position *D* are closer to the sole. However, electrons at position *C* have departed a greater distance from the unperturbed path than have electrons at position *D*. Thus, the electrons have lost a net amount of potential energy, this energy having been transferred to the RF field. The reason for the greater displacement of the electrons moving toward the circuit is that these electrons are in stronger RF fields, since they are closer to the circuit. Electrons at position *G* have moved so far from the unperturbed position that some of them are being intercepted on the circuit. The length from position *A* through position *G* is a half cycle of the electron motion.

Circular *M*-Carcinotron. The *M*-Carcinotrons are generally constructed in the circular reentrant form as shown in Fig. 10-4-3. The slow-wave structure and sole are circular and nearly reentrant to conserve magnet weight. The sole has the appearance of the cathode in a magnetron.

The Litton L-3721 M-BWO, shown in Fig. 10-4-4, is a typical *M*-type backward-wave oscillator (*M*-BWO or Carcinotron) with a minimum power of 200 W at frequency range from 1.0 to 1.4 GHz.

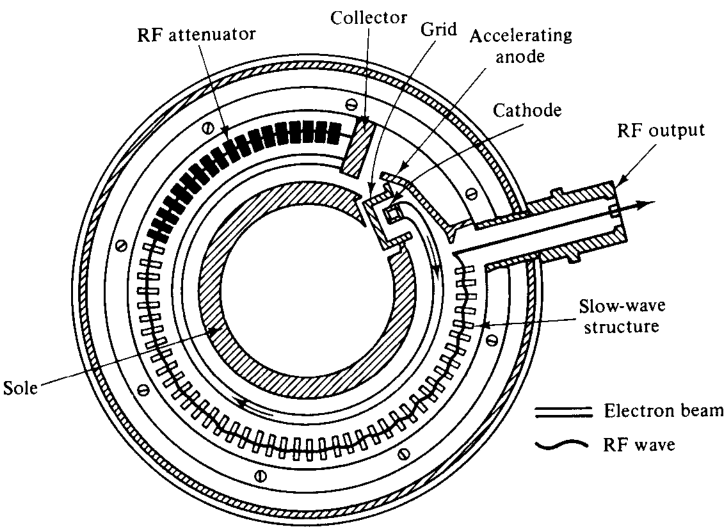


Figure 10-4-3 Schematic diagram of a circular M-Carcinotron. (*Courtesy of Raytheon Company, Microwave Tube Operation.*)

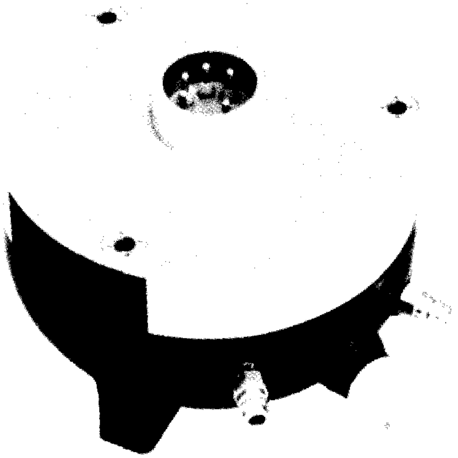


Figure 10-4-4 Photograph of Litton L-3721 BWO. (*Courtesy of Litton Company, Electron Tube Division.*)

In the circular configurations, the delay line is terminated at the collector end by spraying attenuating material on the surfaces of the conductors. The output is taken from the gun end of the delay line which is an interdigital line. Clearly, in this case, the electron drift velocity has to be in synchronism with a backward-space harmonic.

As in the case of O-type devices, the only modification in the secular equation is a change of sign in the circuit equation. If this change is made in Eq. (10-3-1), we write

$$\gamma_0 = j\beta \tag{10-4-1}$$

$$\gamma = jk + \epsilon \tag{10-4-2}$$

By eliminating negligible terms, we obtain for the Carcinotron

$$(\beta^2 - k^2 + j2k\epsilon)\,[j(\beta_e - k) - \epsilon][\beta_m^2 - (\beta_e - k)^2 - j2(\beta_e - k)\epsilon]$$
$$= j\beta\beta_e k^2\left(\beta_e - k + \frac{2\alpha}{1+\alpha^2}\beta_m\right)H_2 \tag{10-4-3}$$

A solution of Eq. (10-4-3) for synchronism can be obtained by setting $\beta = \beta_e$ and $\beta_e - k = \beta_e b'$, where b' is a small number so that terms like b'^2 and $b'\epsilon$ may be neglected. This yields

$$2\epsilon(j\beta_e b' - \epsilon) = \beta\beta_e k\left(\frac{b'}{\beta_m^2} + \frac{2\alpha}{1+\alpha^2}\frac{1}{\beta_m}\right)H^2$$
$$\doteq \beta_e k\frac{2\alpha}{1+\alpha^2}\frac{\beta}{\beta_m}H^2 \tag{10-4-4}$$
$$= 2\beta_e k D^2$$

where

$$D^2 = \frac{\alpha}{1 + \alpha^2} \frac{\beta}{\beta_m} H^2 \tag{10-4-5}$$

$$\epsilon = \beta_e D\delta \tag{10-4-6}$$

$$b' = bD \tag{10-4-7}$$

$$\delta(\delta - jb) = -1 \text{ or } \delta^2 - jb\delta + 1 = 0 \tag{10-4-8}$$

As a result, we have reduced the number of waves to two, with propagation constants given by

$$\gamma_1 = j(\beta_e + b) + \beta_e D\delta_1 \tag{10-4-9}$$

$$\gamma_2 = j(\beta_e + b) + \beta_e D\delta_2 \tag{10-4-10}$$

where the δ's are the roots of Eq. (10-4-8) and they are

$$\delta_1 = j\frac{b - \sqrt{b^2 + 4}}{2}$$

$$\delta_2 = j\frac{b + \sqrt{b^2 + 4}}{2}$$

To determine the amplification of the growing waves, the input reference point is set at $y = 0$, and the output reference point is taken at $y = \ell$. It follows that at $y = 0$, the voltage at the input point can be computed as follows:

$$V_1(0) + V_2(0) = V(0) \tag{10-4-11}$$

$$\frac{V_1(0)}{\delta_1} + \frac{V_2(0)}{\delta_2} = 0 \tag{10-4-12}$$

Solving Eqs. (10-4-11) and (10-4-12) simultaneously we have

$$V_1(0) = \frac{V(0)}{1 - \delta_2/\delta_1} = \frac{\delta_1 V(0)}{\delta_2} \tag{10-4-13}$$

$$V_2(0) = \frac{-V(0)}{1 - \delta_1/\delta_2} = \frac{-\delta_2 V(0)}{\delta_1} \tag{10-4-14}$$

Then the voltage at the output point $y = \ell$ is given by

$$\begin{aligned} V(0) &= V_1(0) \exp(-\gamma_1\ell) + V_2(0) \exp(-\gamma_2\ell) \\ &= V(0)[\delta_1 \exp(-\gamma_1\ell) - \delta_2 \exp(-\gamma_2\ell)]/(\delta_1 - \delta 2) \end{aligned} \tag{10-4-15}$$

The term in the square bracket is the inverse of the voltage gain of the device. Oscillation takes place when this is zero. That is,

$$\delta_1 \exp(-\gamma_1\ell) = \delta_2 \exp(-\gamma_2\ell) \tag{10-4-16}$$

or

$$\delta_1/\delta_2 = \exp(\gamma_1\ell - \gamma_2\ell) \tag{10-4-17}$$

$$= \exp[-\beta_e D\ell(\delta_2 - \delta_2)]$$

From Eq. (10-4-8) we have

$$\delta_1/\delta_2 = \frac{b - \sqrt{b^2 + 4}}{b + \sqrt{b^2 + 4}} \tag{10-4-18}$$

and

$$\delta_2 - \delta_1 = j\sqrt{b^2 + 4} \tag{10-4-19}$$

Then

$$\delta_1/\delta_2 = \exp\left(-j\beta_e D\ell\sqrt{b^2 + 4}\right) \tag{10-4-20}$$

Equations (10-4-18) and (10-4-20) can only be satisfied simultaneously if $b = 0$ and $\delta_1 = -\delta_2$. Then

$$2\beta_e D\ell = (2n + 1)\pi \tag{10-4-21}$$

where n = any integer numbers. If we introduce N as usually defined through $\beta_e\ell = 2\pi N$, the oscillation condition becomes

$$DN = \frac{2n + 1}{4} \tag{10-4-22}$$

Example 10-4-1: Carcinotron Characteristics

A circular carcinotron has the operating parameters:

Anode voltage:	$V_0 = 20$ kV
Anode current:	$I_0 = 3.5$ A
Magnetic flux density:	$B_0 = 0.3$ Wb/m^2
Operating frequency:	$f = 4$ GHz
Characteristic impedance:	$Z_0 = 50\ \Omega$
D factor:	$D = 0.8$
b factor:	$b = 0.5$

Compute:

a. The dc electron velocity
b. The electron-beam phase constant
c. The delta differentials
d. The propagation constants
e. The oscillation condition

Solution

a. The dc electron velocity is

$$\mathcal{V}_0 = 0.593 \times 10^6 \times (20 \times 10^3)^{1/2} = 0.8386 \times 10^8 \text{ m/s}$$

b. The electron-beam phase constant is

$$\beta_e = \frac{\omega}{\mathcal{V}_0} = \frac{2\pi \times 4 \times 10^9}{0.8386 \times 10^8} = 300 \text{ rad/m}$$

c. The delta differentials are

$$\delta_1 = j\frac{0.5 - \sqrt{(0.5)^2 + 4}}{2} = -j0.78$$

$$\delta_2 = j\frac{0.5 + \sqrt{(0.5)^2 + 4}}{2} = j1.28$$

d. The propagation constants are

$$\gamma_1 = j(\beta_e + b) + \beta_e D\delta_1$$
$$= j(300 + 0.5) + 0.5 \times 0.8 \times (-j0.78) = j300.20$$
$$\gamma_2 = j(300 + 0.5) + 0.5 \times 0.8 \times (-j1.28) = j301.00$$

e. The oscillation occurs at

$$DN = 1.25 \qquad \text{for } n = 1$$

then $$N = 1.5625$$

and $$\ell = \frac{2\pi N}{\beta_e} = \frac{2\pi \times 1.5625}{300} = 3.27 \text{ cm}$$

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PROBLEMS

Magnetrons

10-1. Describe the principle of operation for a normal cylindrical magnetron and its characteristics.

10-2. A normal cylindrical magnetron has the following parameters:

Inner radius:	$R_a = 0.15$ meter
Outer radius:	$R_b = 0.45$ meter
Magnetic flux density:	$B_0 = 1.2$ milliwebers/m^2

a. Determine the Hull cutoff voltage.
b. Determine the cutoff magnetic flux density if the beam voltage V_0 is 6000 V.

10-3. It is assumed that in a normal cylindrical magnetron the inner cylinder of radius a carries a current of I_0 in the z direction (i.e., $\mathrm{I} = I_0 u_z$) and the anode voltage is V_0. The outer radius is b. Determine the differential equation in terms of the anode voltage V_0 and the current I_0.

10-4. Compare the cutoff conditions for an inverted cylindrical magnetron (i.e., the inner cathode voltage is V_0 and the outer anode is grounded) with a normal cylindrical magnetron. It is assumed that the magnetic field does no work on the electrons.

10-5. It is assumed that electrons in an inverted cylindrical magnetron leave the interior of the coaxial cathode with initial velocity caused by thermal voltage V_t in volts. Find the initial velocity required for the electrons to just hit the anode at the center conductor.

10-6. It is assumed that electrons in an inverted cylindrical magnetron leave the interior of the coaxial cathode with zero initial velocity. Find the minimum velocity for an electron to just graze the anode at the center conductor.

10-7. In a linear magnetron the electric and magnetic field intensities as shown in Fig. P10-7 are given by

$$\mathbf{E} = E_2\mathbf{u}_z = -\frac{V_0}{d}\mathbf{u}_z$$

$$\mathbf{B} = B_y\mathbf{u}_y$$

Determine the trajectory of an electron with an initial velocity $\mathcal{V}_0$ in the z direction.

$$\mathbf{E} = E_2\mathbf{u}_z = -\frac{V_0}{d}\mathbf{u}_z$$

$$\mathbf{B} = B_y\mathbf{u}_y$$

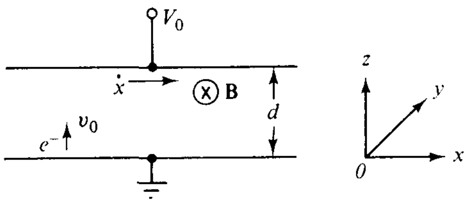

Figure P10-7

10-8. An X-band pulsed cylindrical magnetron has the following parameters:

Anode voltage:	$V_0 = 32$ kV
Anode current:	$I_0 = 84$ A
Magnetic flux density:	$B_0 = 0.01$ Wb/m^2
Radius of cathode cylinder:	$a = 6$ cm
Radius of vane edge to center:	$b = 12$ cm

Compute:

a. The cyclotron angular frequency
b. The cutoff voltage for a fixed B_0
c. The cutoff magnetic flux density for a fixed V_0

10-9. An X-band pulsed conventional magnetron has the following parameters:

Anode voltage:	$V_0 = 22$ kV
Anode current:	$I_0 = 28$ A
Operating frequency:	$f = 10$ GHz
Resonator conductance:	$G_r = 3 \times 10^{-4}\ \mho$
Loaded conductance:	$G_\ell = 3 \times 10^{-5}\ \mho$
Vane capacitance:	$C = 3$ pF
Duty cycle:	$DC = 0.001$
Power loss:	$P_{\text{loss}} = 200$ kW

Compute:

a. The angular resonant frequency
b. The unloaded quality factor Q_{un}
c. The loaded quality factor Q_ℓ
d. The external quality factor Q_{ex}
e. The circuit efficiency
f. The electronic efficiency

10-10. A linear magnetron has the following parameters:

Anode voltage:	$V_0 = 20$ kV
Anode current:	$I_0 = 17$ A
Magnetic flux density:	$B_0 = 0.01$ Wb/m^2
Distance between cathode and anode:	$d = 6$ cm

Calculate:

a. The Hull cutoff voltage for a fixed B_0
b. The Hull cutoff magnetic flux density for a fixed V_0

10-11. A linear magnetron has the following parameters:

Anode voltage:	$V_0 = 32$ kV
Anode current:	$I_0 = 60$ A
Operating frequency:	$f = 10$ GHz

Magnetic flux density:	$B_0 = 0.01$ Wb/m^2
Hub thickness:	$h = 3$ cm
Distance between anode and cathode:	$d = 6$ cm

Compute:
a. The electron velocity at the hub surface
b. The phase velocity for synchronism
c. The Hartree anode voltage

10-12. An inverted coaxial magnetron has the following parameters:

Anode voltage:	$V_0 = 30$ kV
Cathode current:	$I_0 = 25$ A
Anode radius:	$a = 2.5$ cm
Cathode radius:	$b = 5$ cm
Magnetic flux density:	$B_0 = 0.01$ Wb/m^2

Determine:
a. The cutoff voltage for a fixed B_0
b. The cutoff magnetic flux density for a fixed V_0

10-13. A frequency-agile coaxial magnetron has the following parameters:

Pulse duration:	$\tau = 0.30, 0.60, 0.90\ \mu$s
Duty cycle:	$DC = 0.0011$
Pulse rate on target:	$N = 15$ per scan

Determine:
a. The agile excursion
b. The pulse-to-pulse frequency separation
c. The signal frequency
d. The time for N pulse
e. The agile rate

Crossed-Field Amplifiers (CFAs)

10-14. Derive Eq. (10-2-4).

10-15. Derive the circuit-efficiency Eq. (10-2-5).

10-16. Derive Eqs. (10-2-8), (10-2-10), and (10-2-11).

10-17. A CFA has the following operating parameters:

Anode dc voltage:	$V_{a0} = 3$ kV
Anode dc current:	$I_{a0} = 3$ A
Electronic efficiency:	$\eta_e = 25\%$
RF input power:	$P_{in} = 100$ W

Compute:
a. The induced RF power

b. The total RF output power
c. The power gain in dB

10-18. A CFA operates under the following parameters:

Anode dc voltage:	$V_{a0} = 1.80$ kV
Anode dc current:	$I_{a0} = 1.30$ A
Electronic efficiency:	$\eta_e = 22\%$
RF input power:	$P_{in} = 70$ W

Calculate:
a. The induced RF power
b. The total RF output power
c. The power gain in dB

Amplitrons

10-19. An Amplitron operates under the following parameters:

Operating frequency:	$f = 9$ GHz
Anode voltage:	$V_0 = 20$ kV
Anode current:	$I_0 = 3.5$ A
Magnetic flux density:	$B_0 = 0.3$ Wb/m^2
Characteristic impedance:	$Z_0 = 50\ \Omega$

Compute:
a. The dc electron-beam velocity
b. The electron-beam phase constant
c. The cyclotron angular frequency
d. The cyclotron phase constant
e. The gain parameter

10-20. An Amplitron has the following operating parameters:

Operating frequency:	$f = 10$ GHz
Anode voltage:	$V_0 = 25$ kV
Anode current:	$I_0 = 4$ A
Magnetic flux density:	$B_0 = 0.35$ Wb/m^2
Characteristic impedance:	$Z_0 = 50\ \Omega$

Calculate:
a. The dc electron-beam velocity
b. The electron-beam phase constant
c. The cyclotron angular frequency
d. The cyclotron phase constant
e. The gain parameter

Carcinotrons

10-21. A circular Carcinotron operates under the following parameters:

Operating frequency:	$f = 8$ GHz
Anode voltage:	$V_0 = 25$ kV
Anode current:	$I_0 = 4$ A
Magnetic flux density:	$B_0 = 0.35$ Wb/m^2
Characteristic impedance:	$Z_0 = 50$ ohms
D factor:	$D = 0.75$
b factor:	$b = 0.50$

Calculate:

a. The dc electron velocity
b. The electron-beam phase constant
c. The delta differentials
d. The propagation constants
e. The oscillation condition